IDENTIFICATION WITH FINITELY MANY DATA POINTS: THE LSCR APPROACH

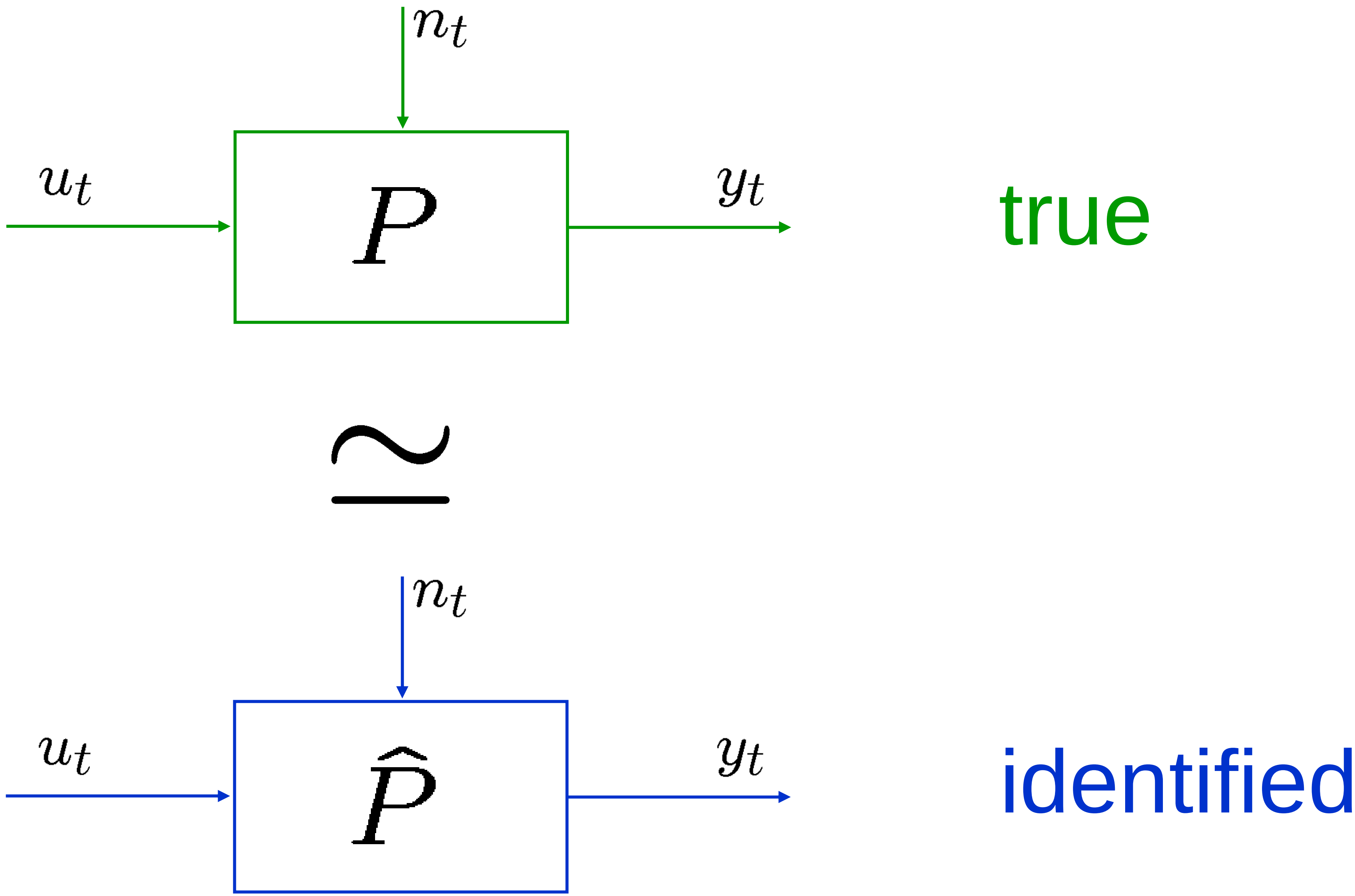
Marco Campi

University of Brescia

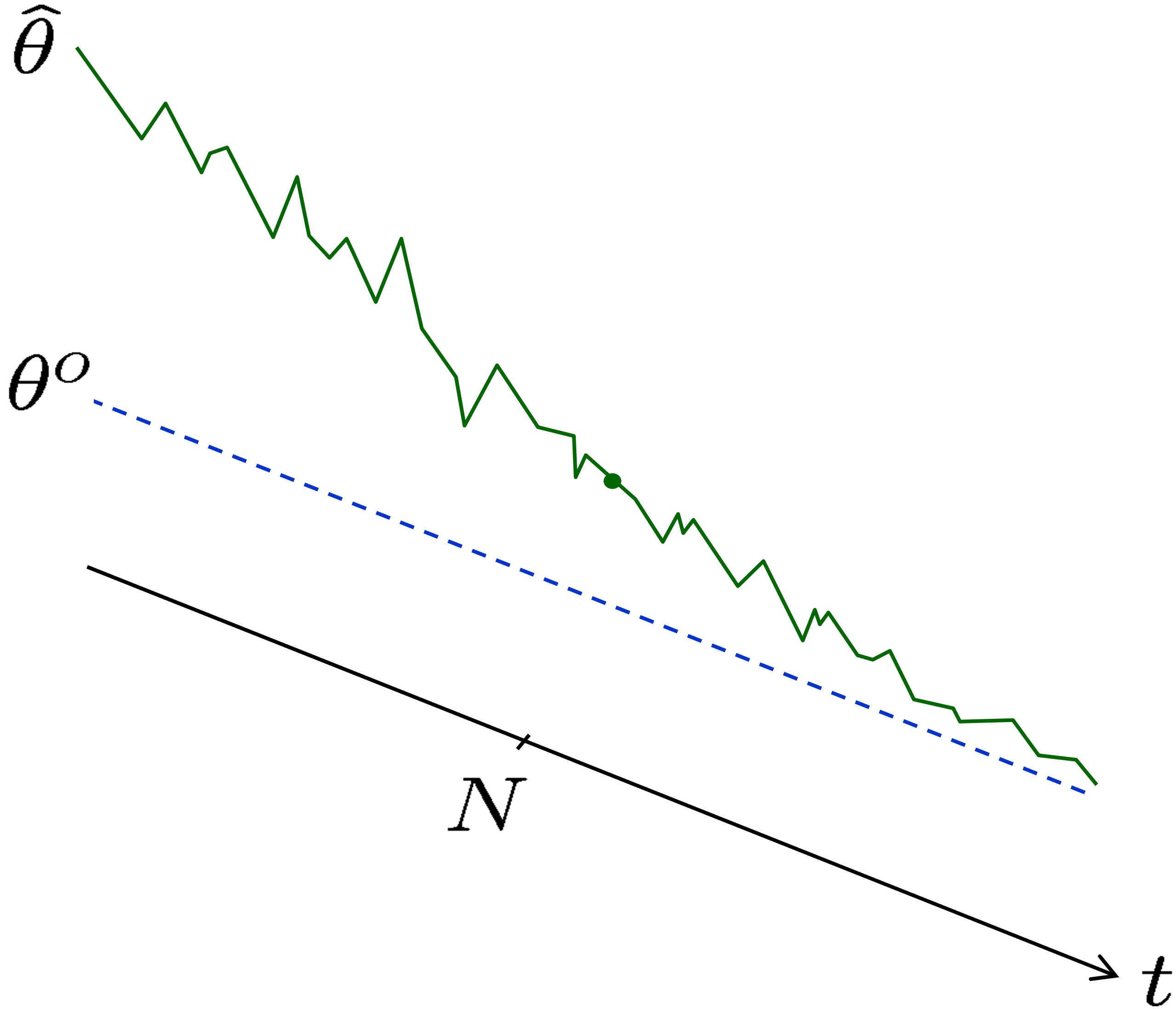
Erik Weyer

University of Melbourne

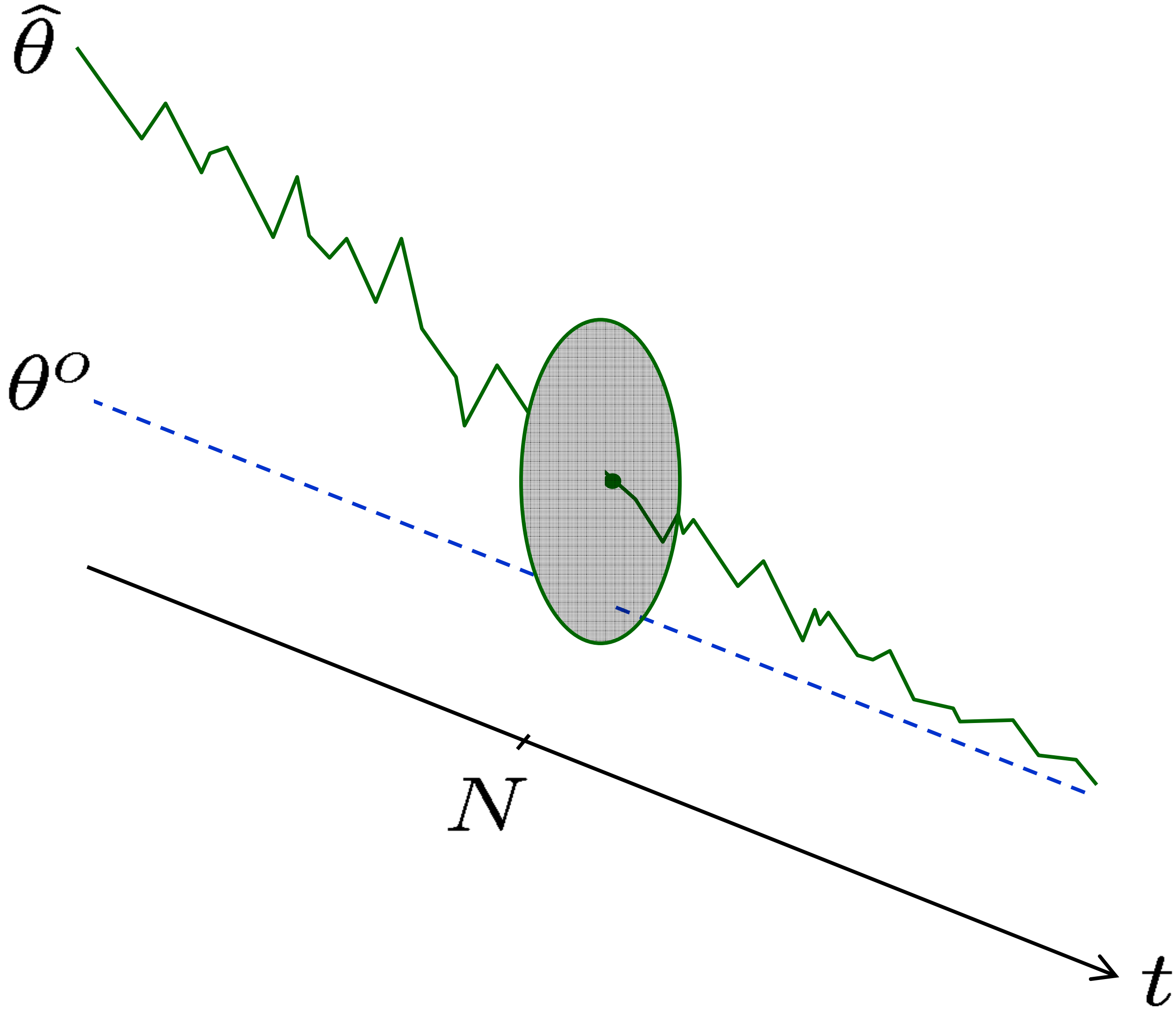
PART I: From nominal-model to model-set identification



n_t
u_t
P
y_t
true
$\simeq$
n_t
u_t
$\widehat{P}$
y_t
identified



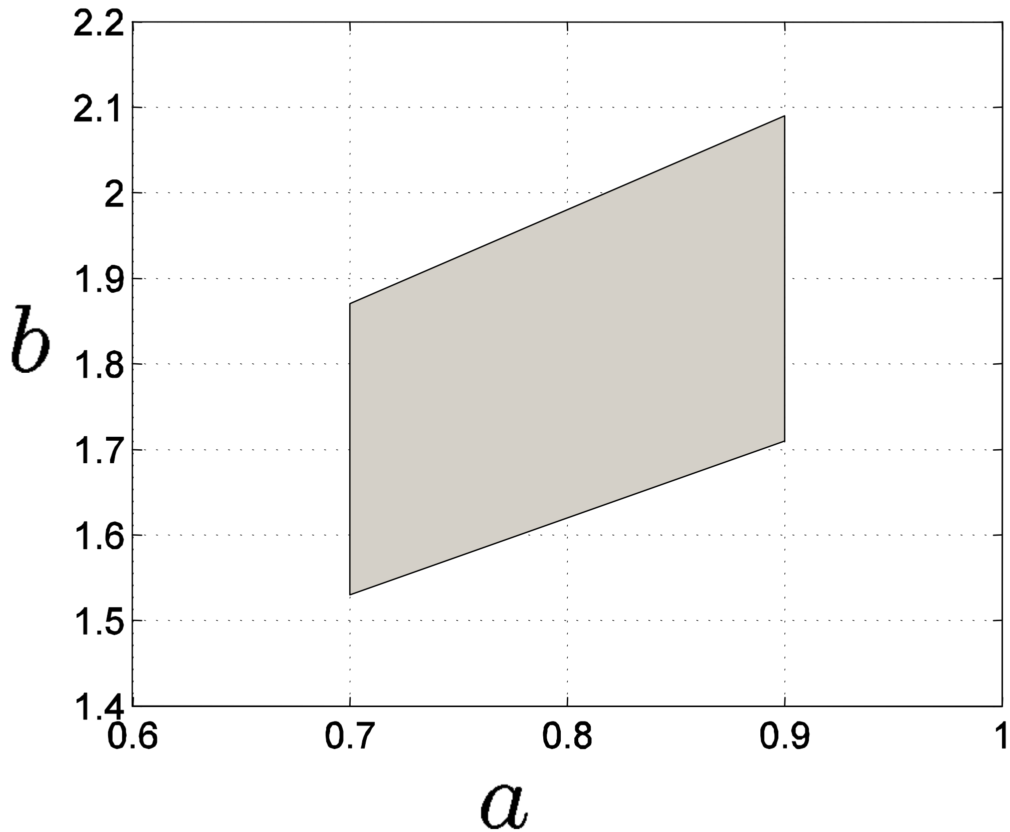
$$Pr\{\theta^O = \widehat{\theta}\} = 0$$

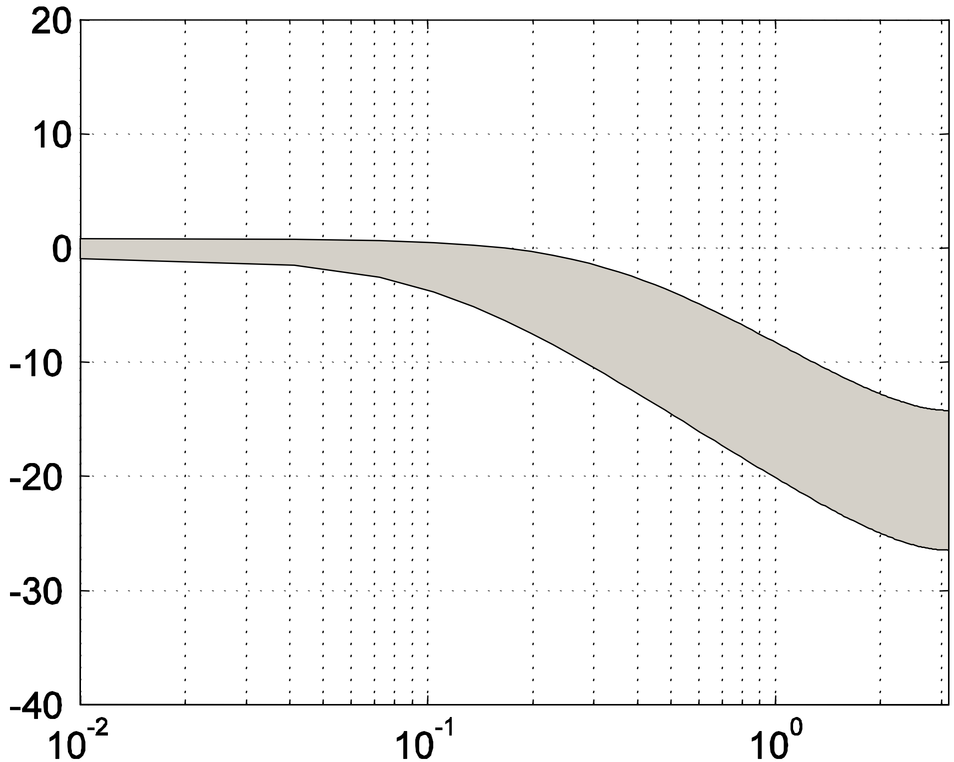


$$Pr\{\theta^o \in region\} \geq 0$$

Example

$$y_t = a^o y_{t-1} + b^o u_{t-1} + n_t$$





20
10
0
-10
-20
-30
-40
10^{-2}
10^{-1}
10^{0}

Goal: finding confidence regions, guaranteed under general assumptions

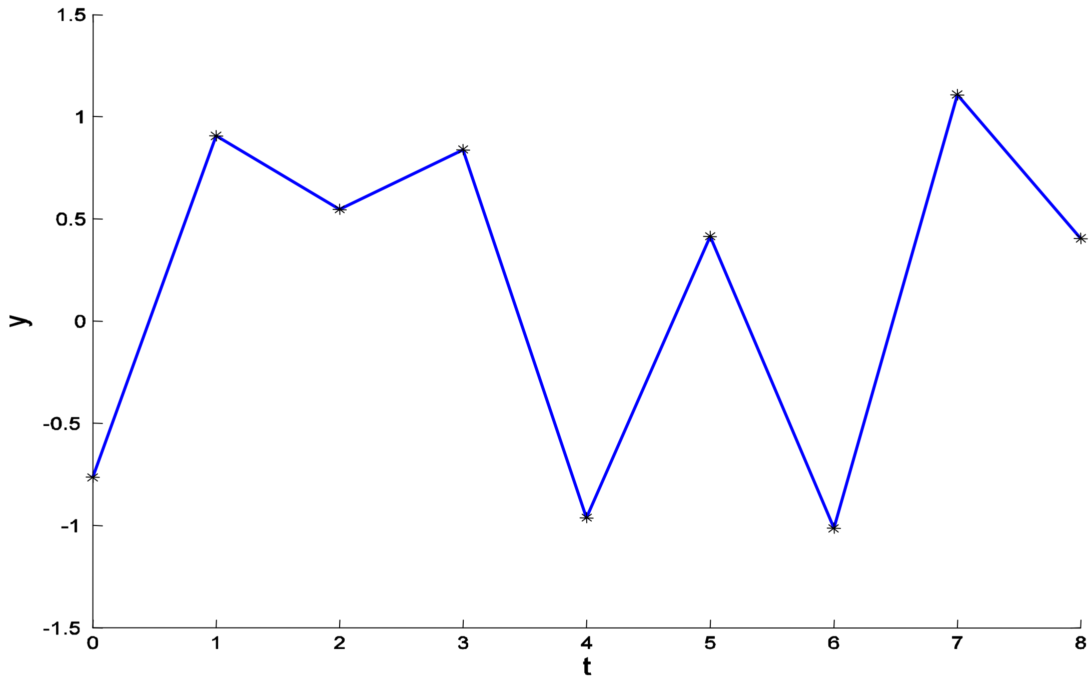
PART II:

LSCR = Leave-out Sign-dominant Correlation Regions

A simple Example

$$y_t + a^o y_{t-1} = w_t$$

w_t = independent, symmetrically distributed



Find a “guaranteed” interval for a^o

The LSCR approach

$$y_t + ay_{t-1} = w_t$$

$$\widehat{y}_t = -ay_{t-1}$$

$$\epsilon_t(a) = y_t - \widehat{y}_t = y_t + ay_{t-1}$$

$$\frac{1}{N}\sum_{t=1}^{N} \epsilon_{t-1}(a)\epsilon_t(a)$$

empirical correlation

this is a function of a

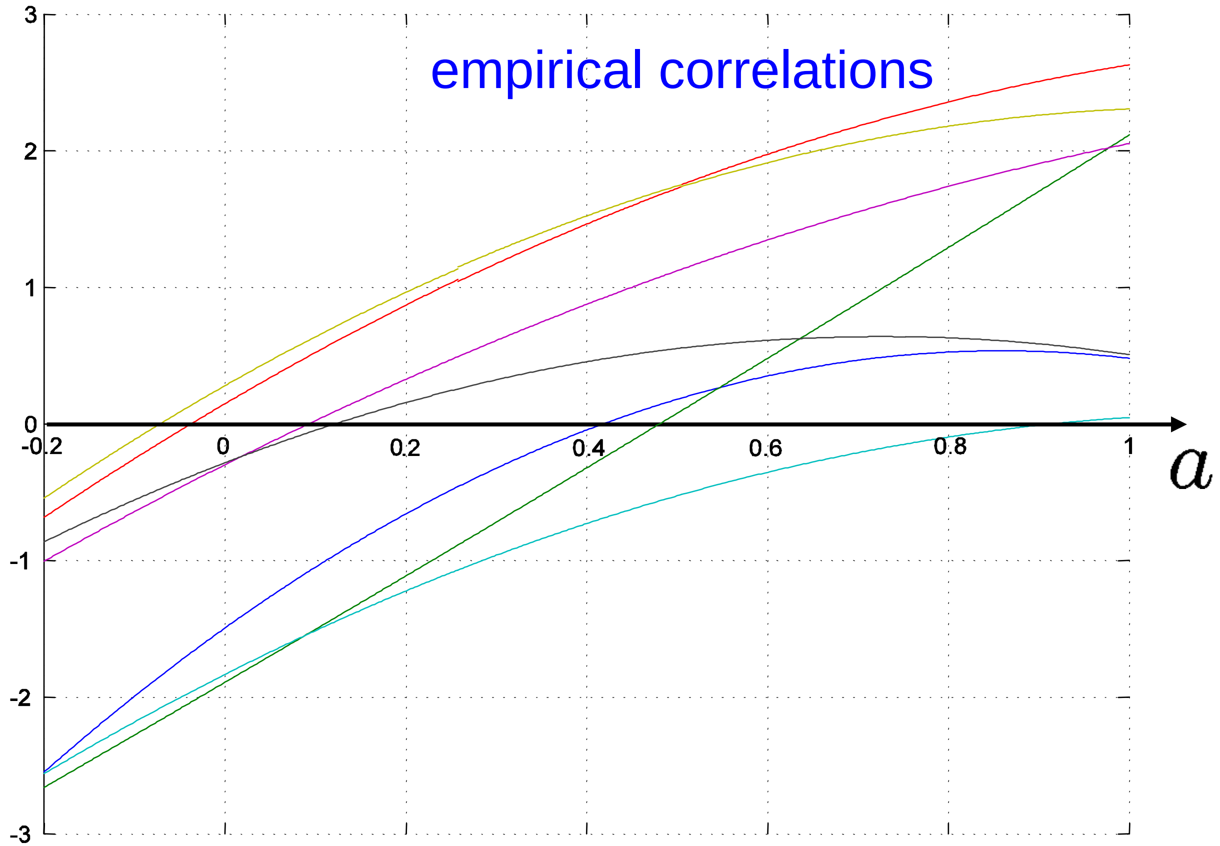
	1	2	3	4	5	6	7
I_1	●	●	○	●	●	○	○
I_2	●	○	●	●	○	●	○
I_3	○	●	●	○	●	●	○
I_4	●	●	○	○	○	●	●
I_5	●	○	●	○	●	○	●
I_6	○	●	●	●	○	○	●
I_7	○	○	○	●	●	●	●

$$\Longrightarrow \frac{1}{4}[\epsilon_1(a)\epsilon_2(a) + \epsilon_2(a)\epsilon_3(a) + \epsilon_4(a)\epsilon_5(a) + \epsilon_5(a)\epsilon_6(a)]$$

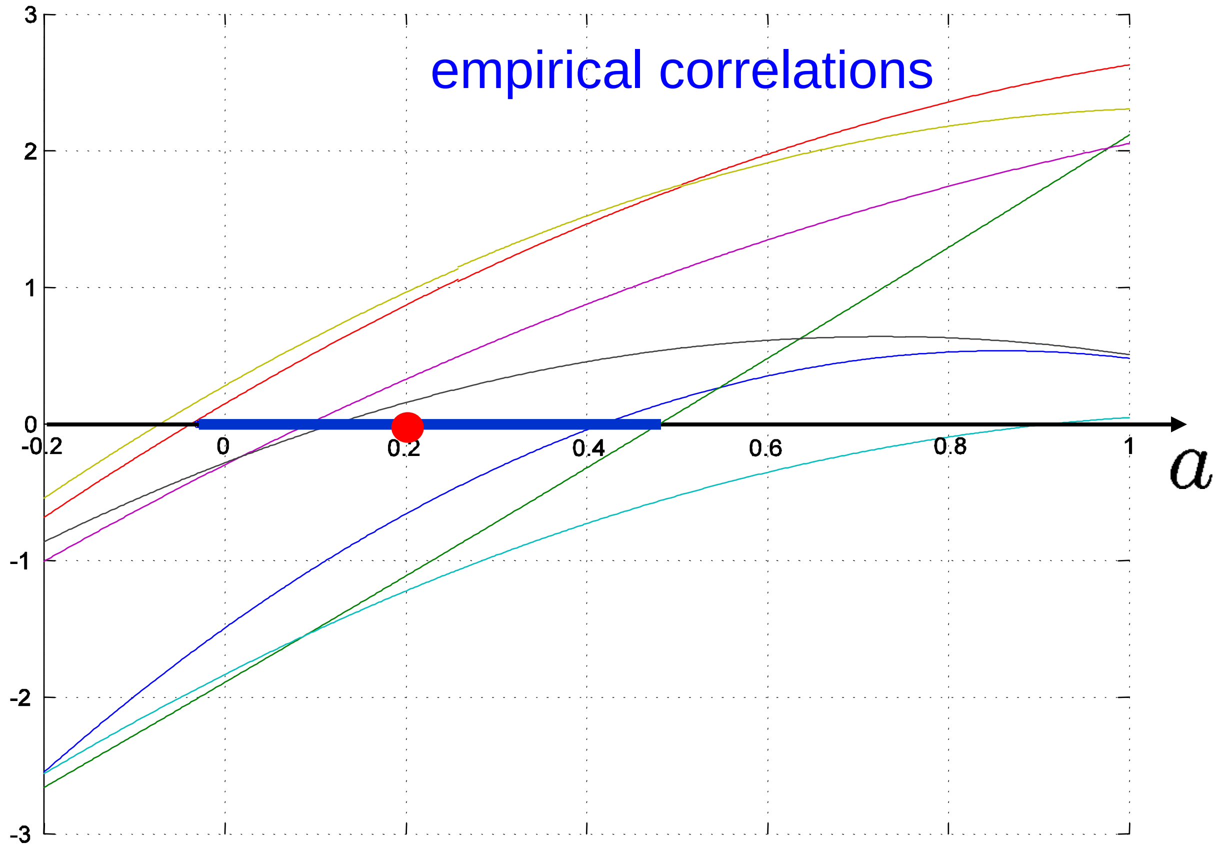
$$\Longrightarrow \frac{1}{4}[\epsilon_1(a)\epsilon_2(a) + \epsilon_3(a)\epsilon_4(a) + \epsilon_4(a)\epsilon_5(a) + \epsilon_6(a)\epsilon_7(a)]$$

$$\Longrightarrow \frac{1}{4}[\epsilon_2(a)\epsilon_3(a) + \epsilon_3(a)\epsilon_4(a) + \epsilon_5(a)\epsilon_6(a) + \epsilon_6(a)\epsilon_7(a)]$$

$$\vdots$$



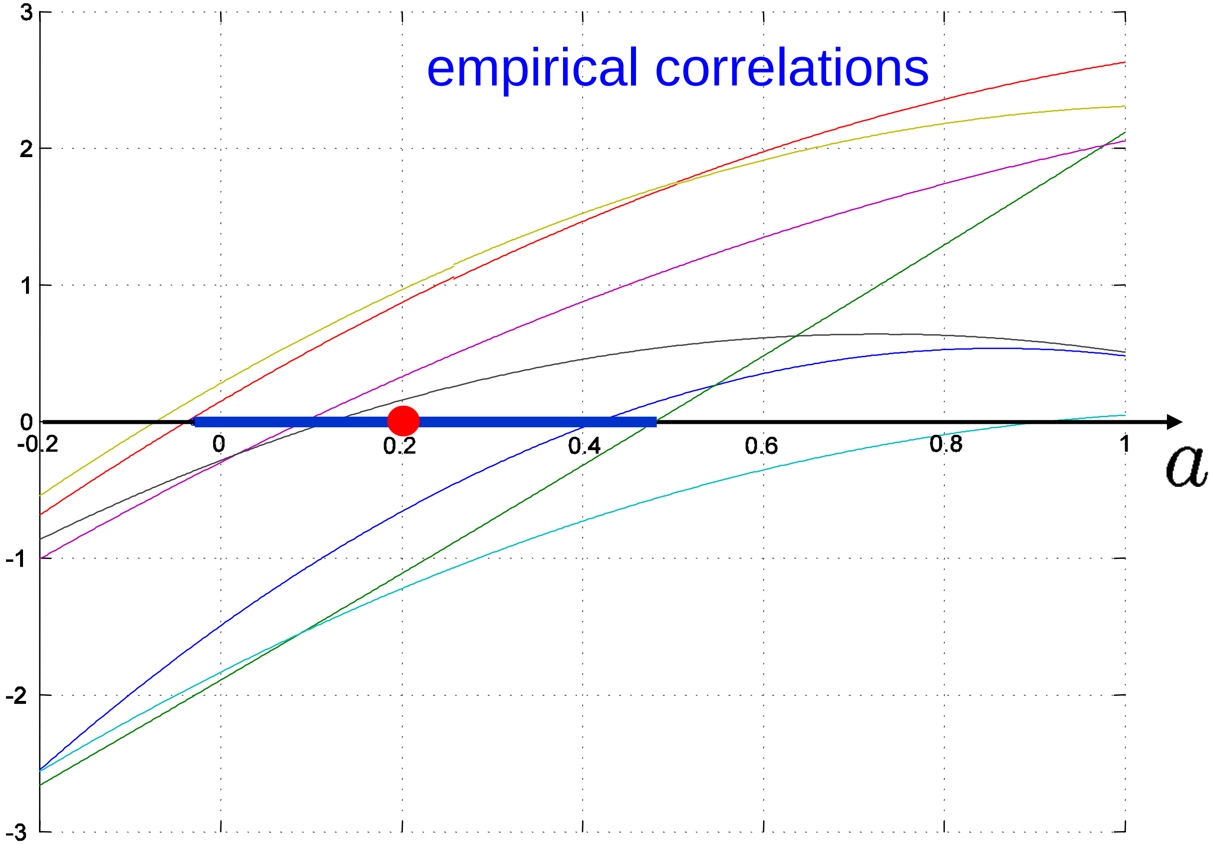
$$\frac{1}{4}\sum_{\{t=1,2,4,5\}}\epsilon_{t-1}(a^o)\epsilon_t(a^o)=\frac{1}{4}\sum_{\{t=1,2,4,5\}}w_{t-1}w_t$$



discard regions where empirical correlations are positive or negative “too many times” (LSCR = Leave-out Sign-dominant Correlation Regions)

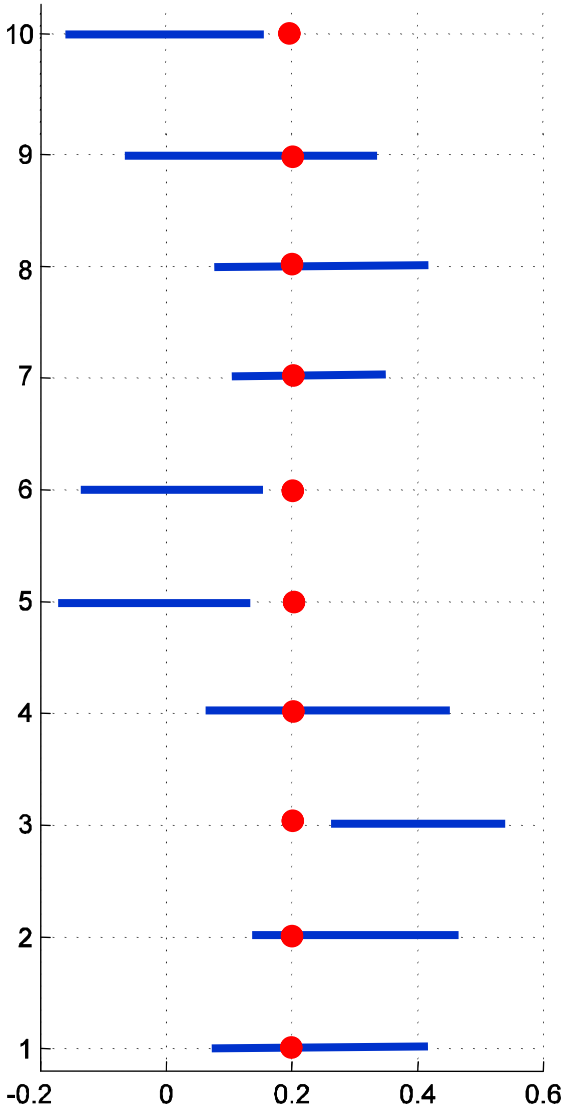
Theorem ('pivotal' result)

For $a = a^o$, the value 0 is "at top", "as second top", ... with the same probability 1/8, independently of the noise characteristics.

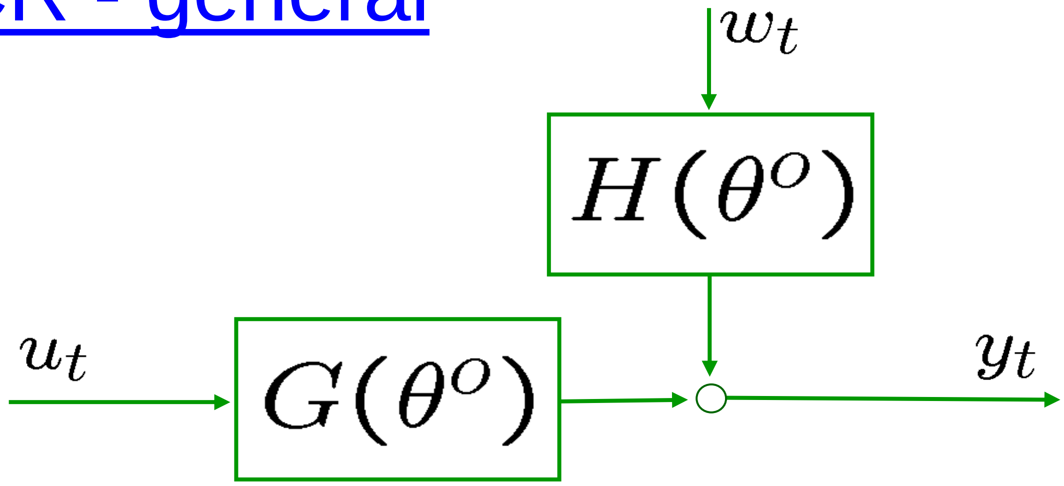


$$Pr\{a^o \in interval\} = 0.5$$

10 more trials

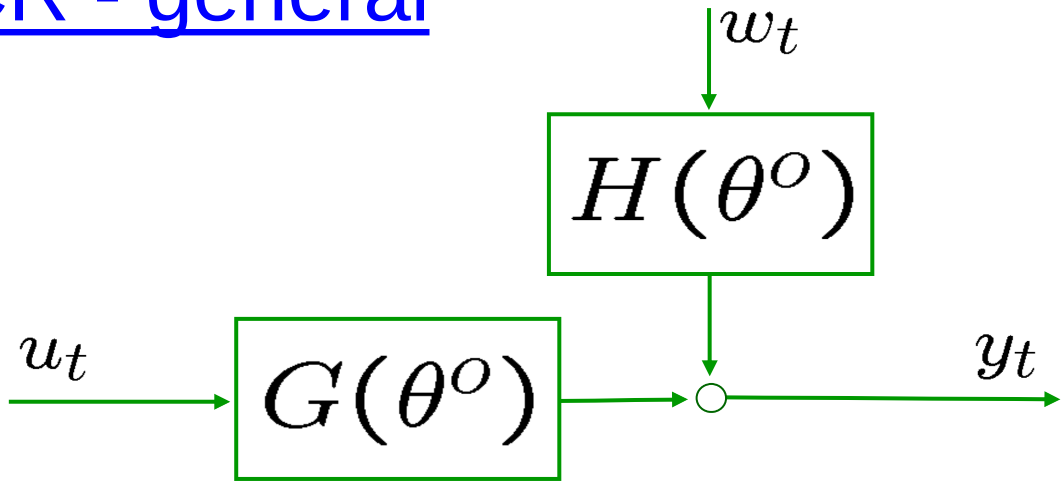


LSCR - general

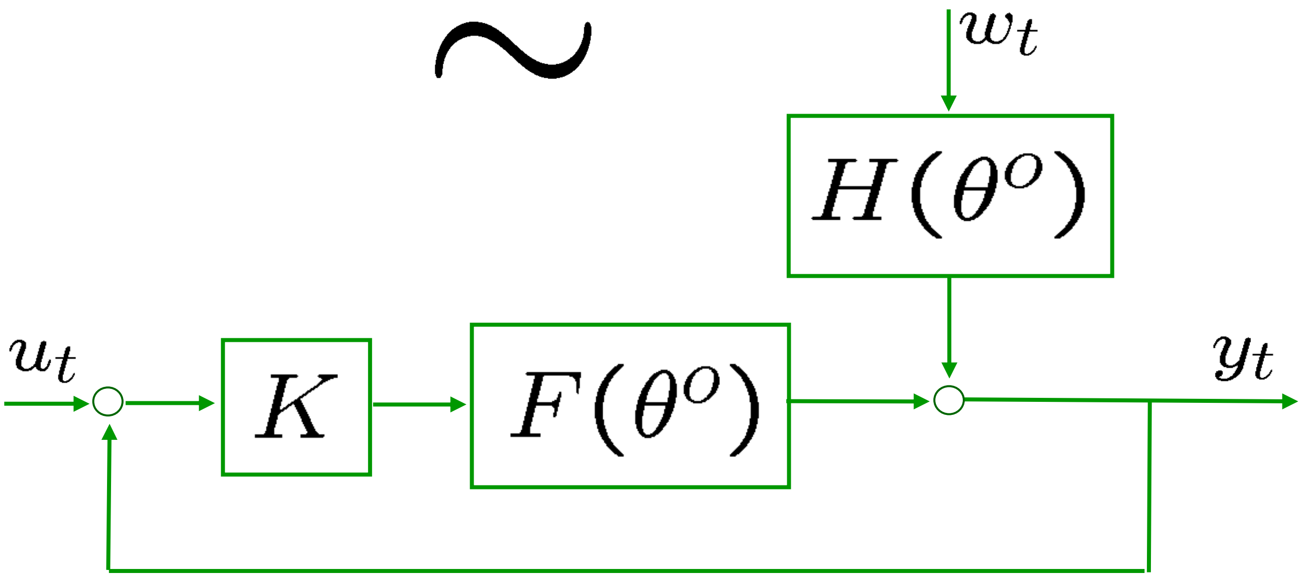


w_t independent of u_t

LSCR - general



$$\sim$$



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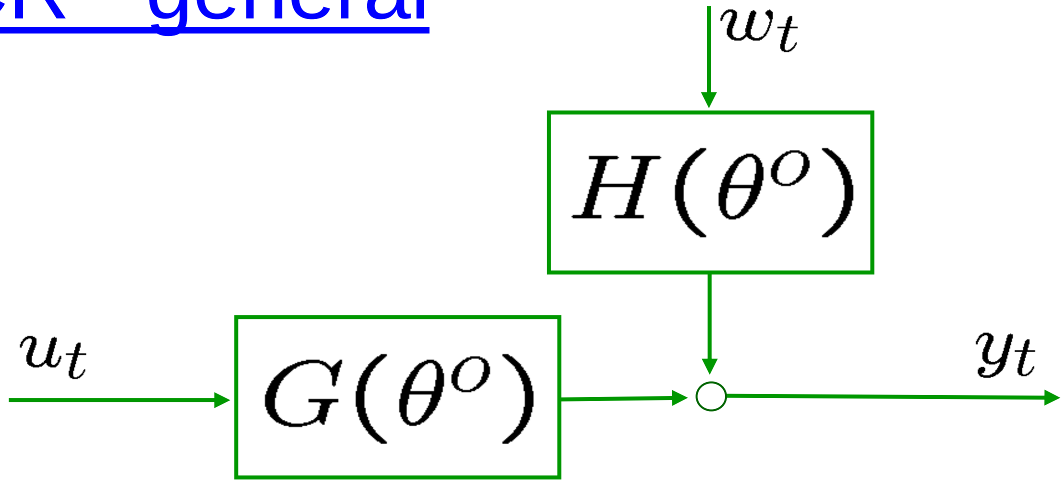

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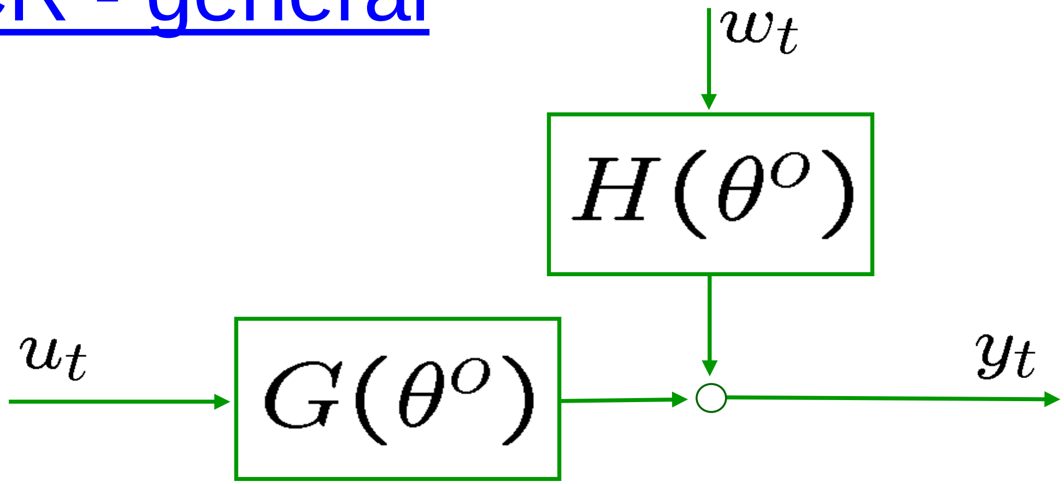
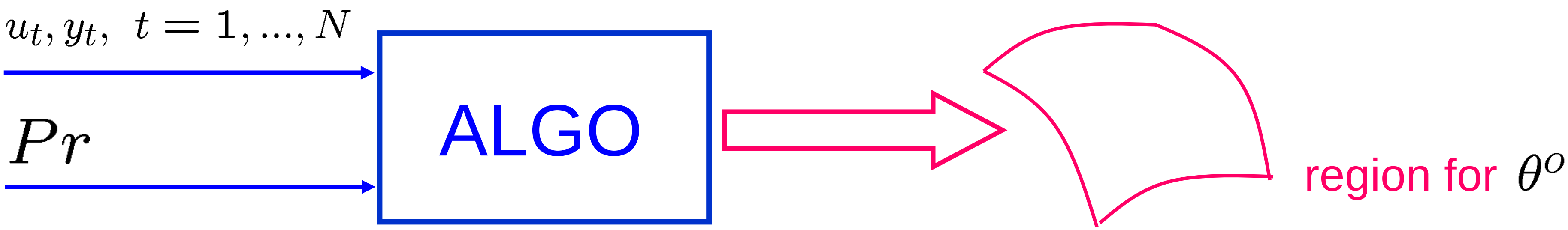
LSCR - general



w_t independent of u_t



region guaranteed under general assumptions on noise

LSCR - general

- $\epsilon_{t-d}(\theta)\epsilon_t(\theta)$
- $u_{t-d}\epsilon_t(\theta)$

Theorem

w_t zero mean, symmetrically distributed
(no assumption on strength of noise)

$$Pr\{\theta^o \in \text{LSCR region}\} = 1 - \frac{q}{M+1} \quad \blacksquare$$

- usually, intersect more regions

Example 1

$$y_t + a^0 y_{t-1} = w_t + c^0 w_{t-1}$$

$$a^0 = -0.5, \quad c^0 = 0.2, \quad w_t \sim WGN(0,1)$$

$N = 1025$

$\epsilon_{t-1}(a,c)\epsilon_t(a,c)$

$\epsilon_{t-2}(a,c)\epsilon_t(a,c)$

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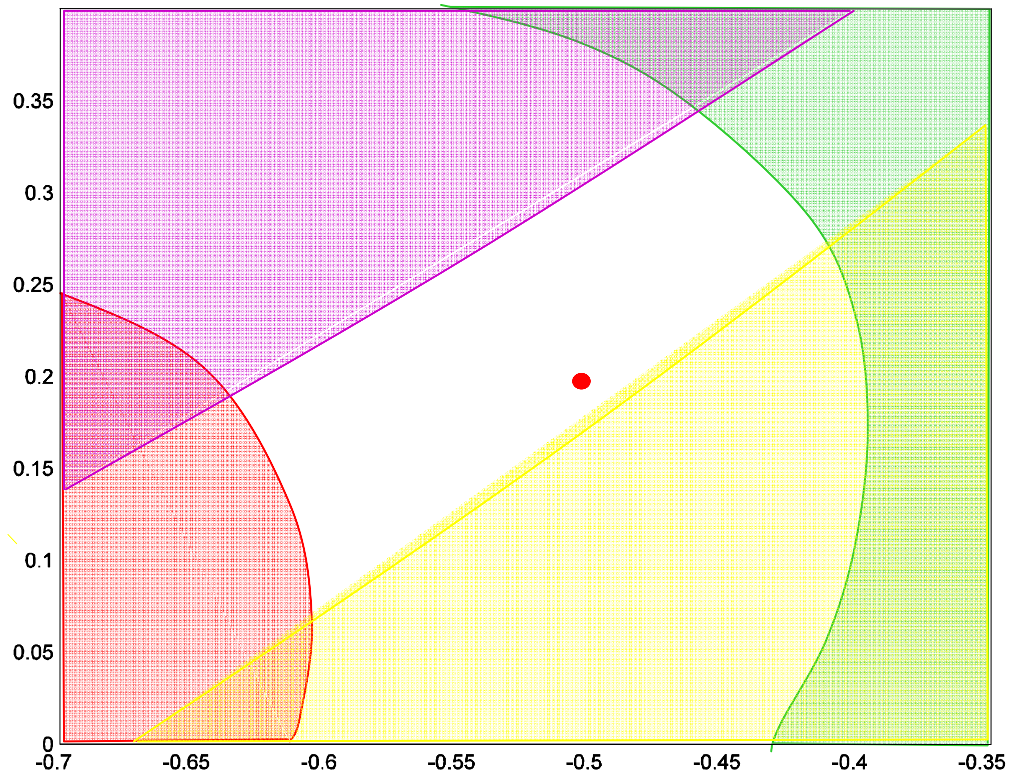
$Pr \geq 95\%$



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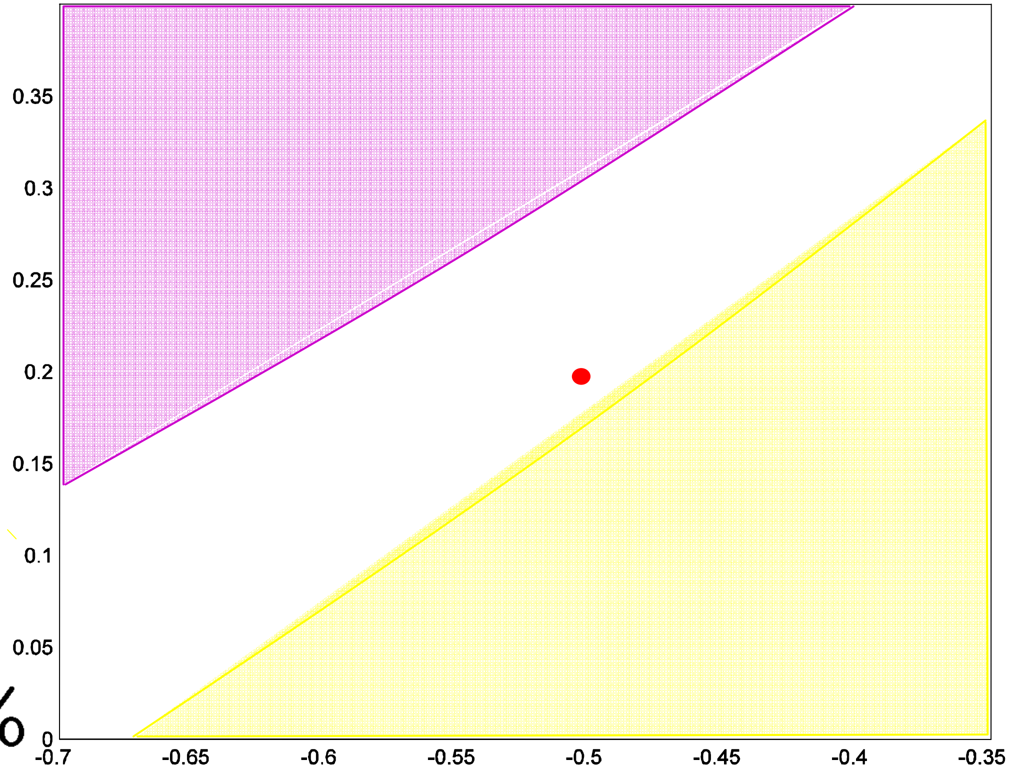
$$\boxed{\epsilon_{t-1}(a,c)\epsilon_t(a,c)}$$

$$\epsilon_{t-2}(a,c)\epsilon_t(a,c)$$



$$Pr = 97.66\%$$

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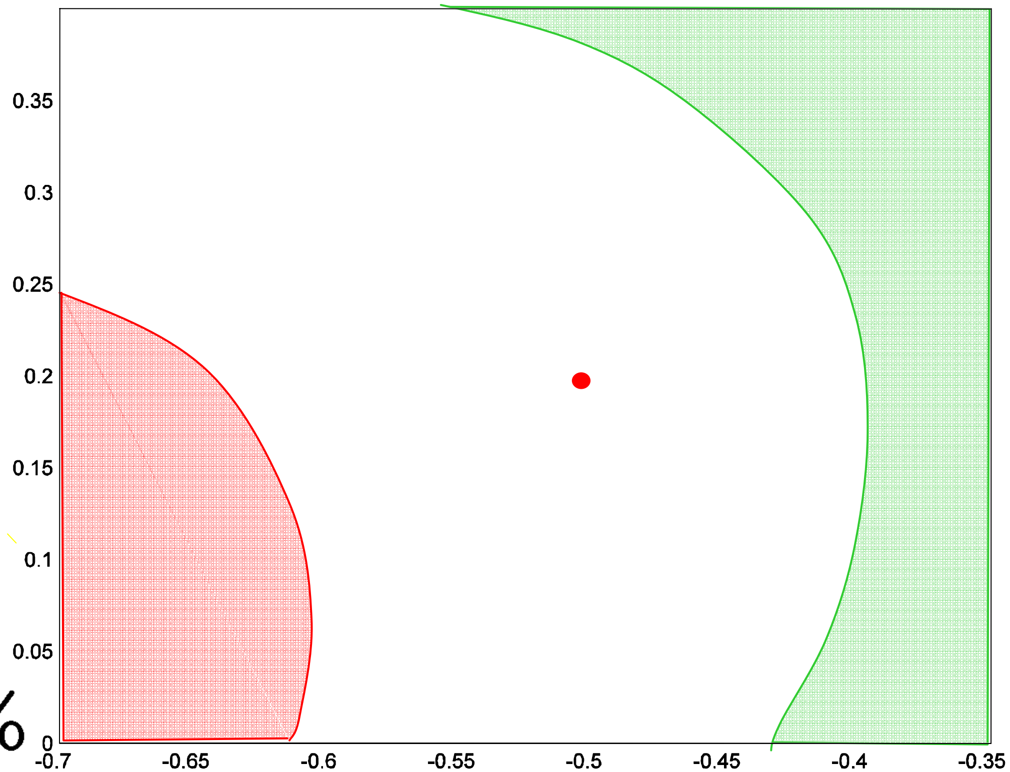
$\boxed{\epsilon_{t-2}(a,c)\epsilon_t(a,c)}$



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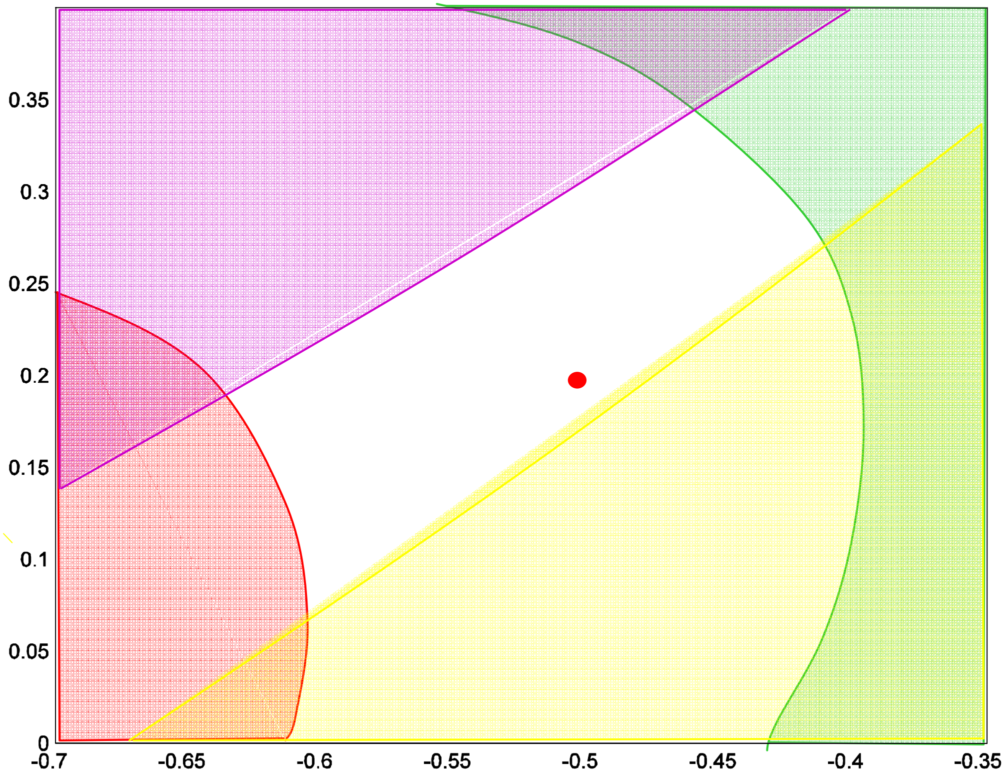

$$Pr \geq 95\%$$

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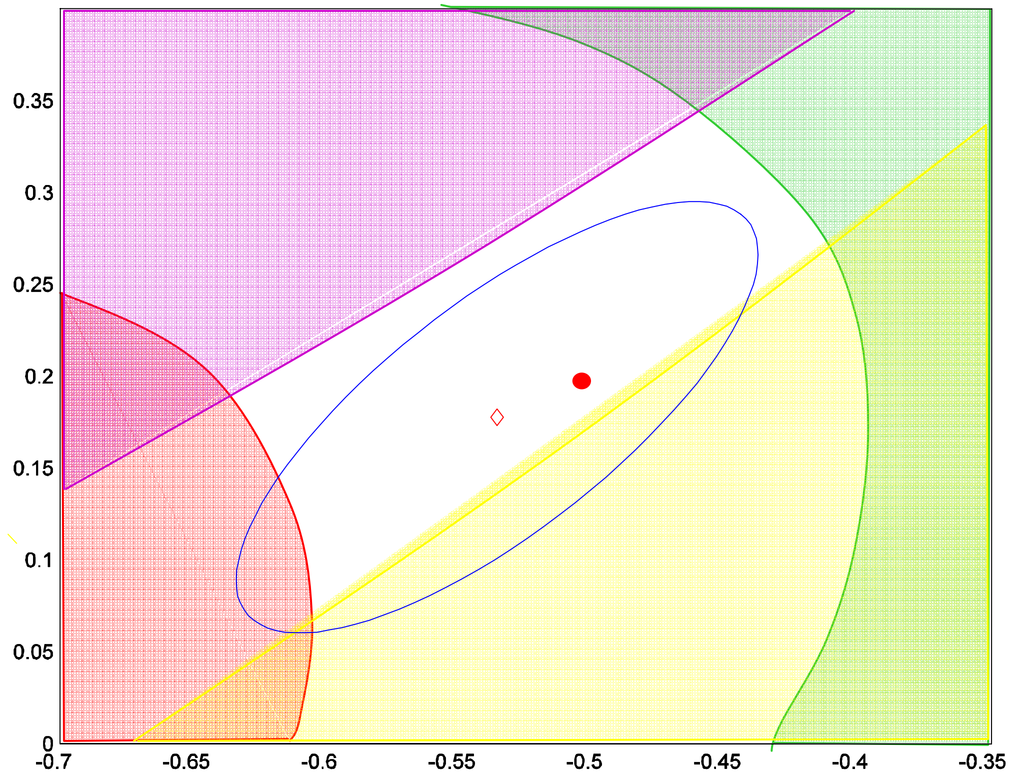
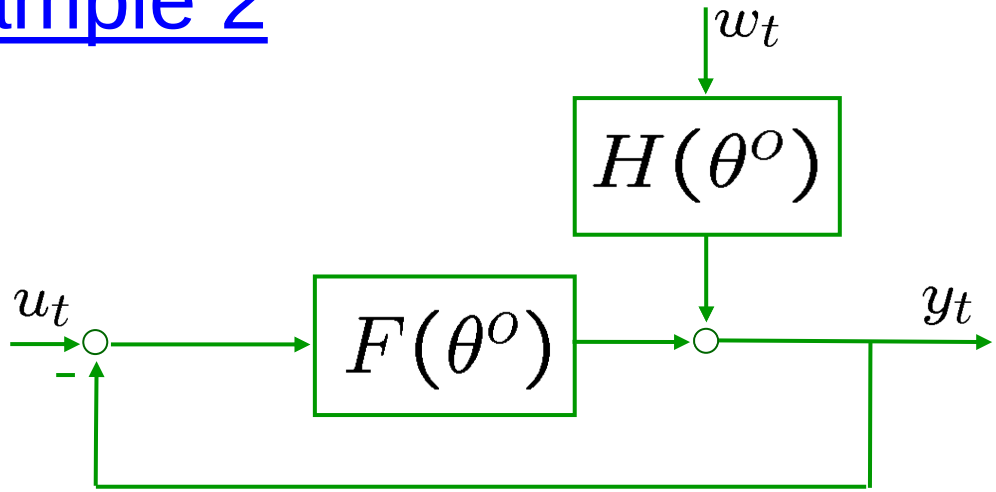


Example 2



$$F^0 = \frac{b^o z^{-1}}{1+a^o z^{-1}} \qquad a^o = -0.7,\ b^o = 0.3$$

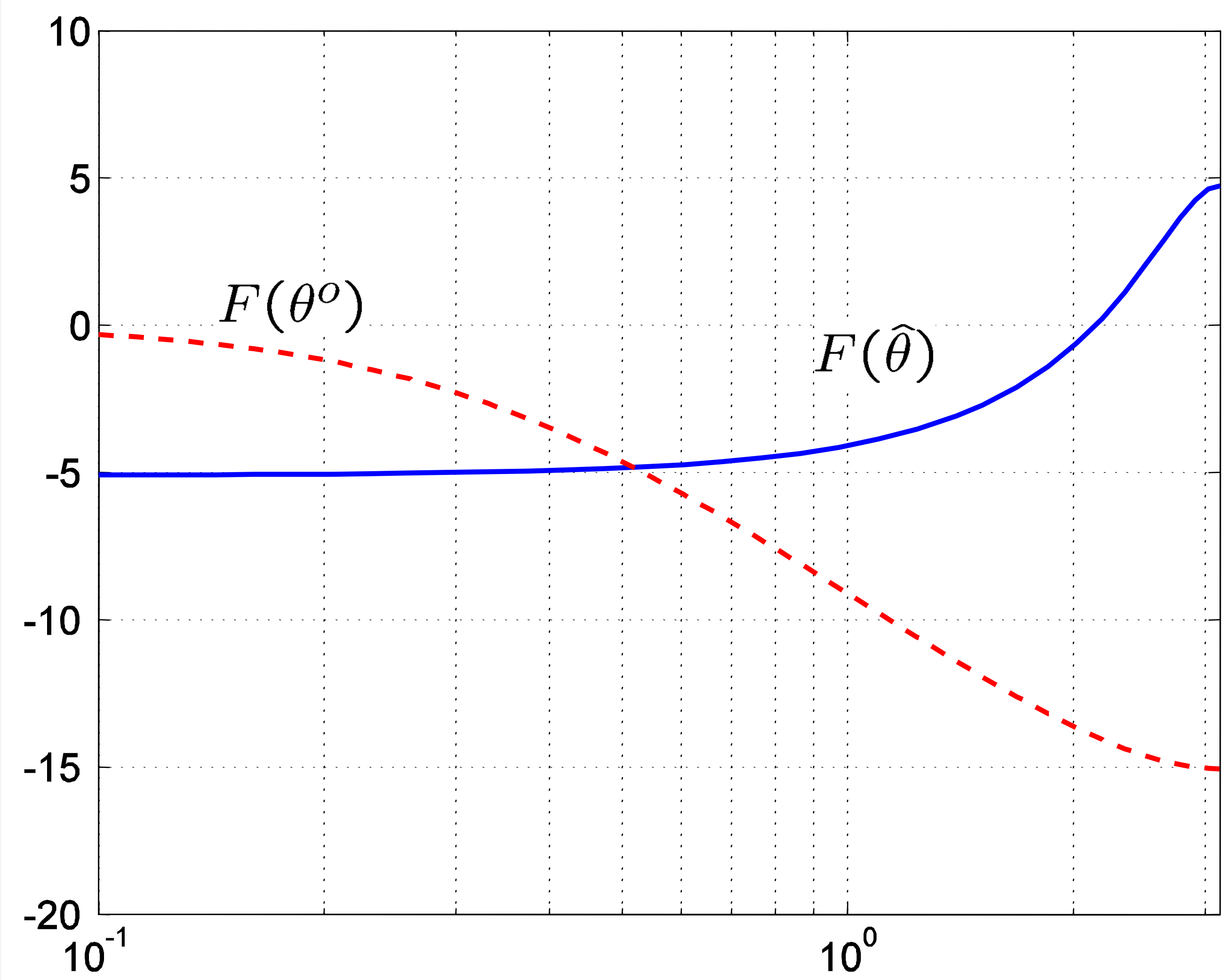
$$H^0 = 1 + h^o z^{-1} \qquad h^o = 0.5$$

$$N = 2047$$

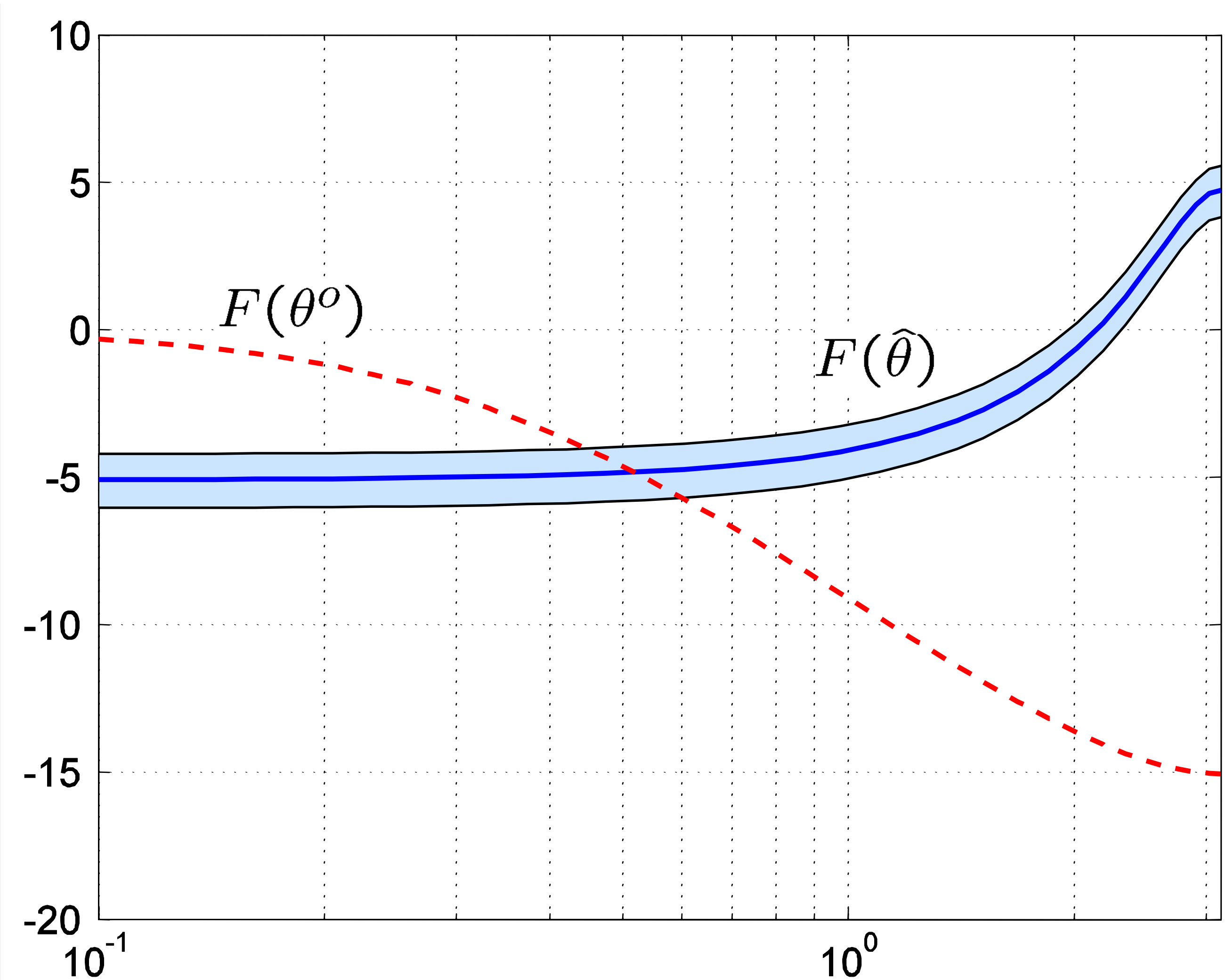
$$w_t \sim WGN(0,1)$$

$$u_t \sim WGN(0,10^{-6})$$

... use asymptotic theory

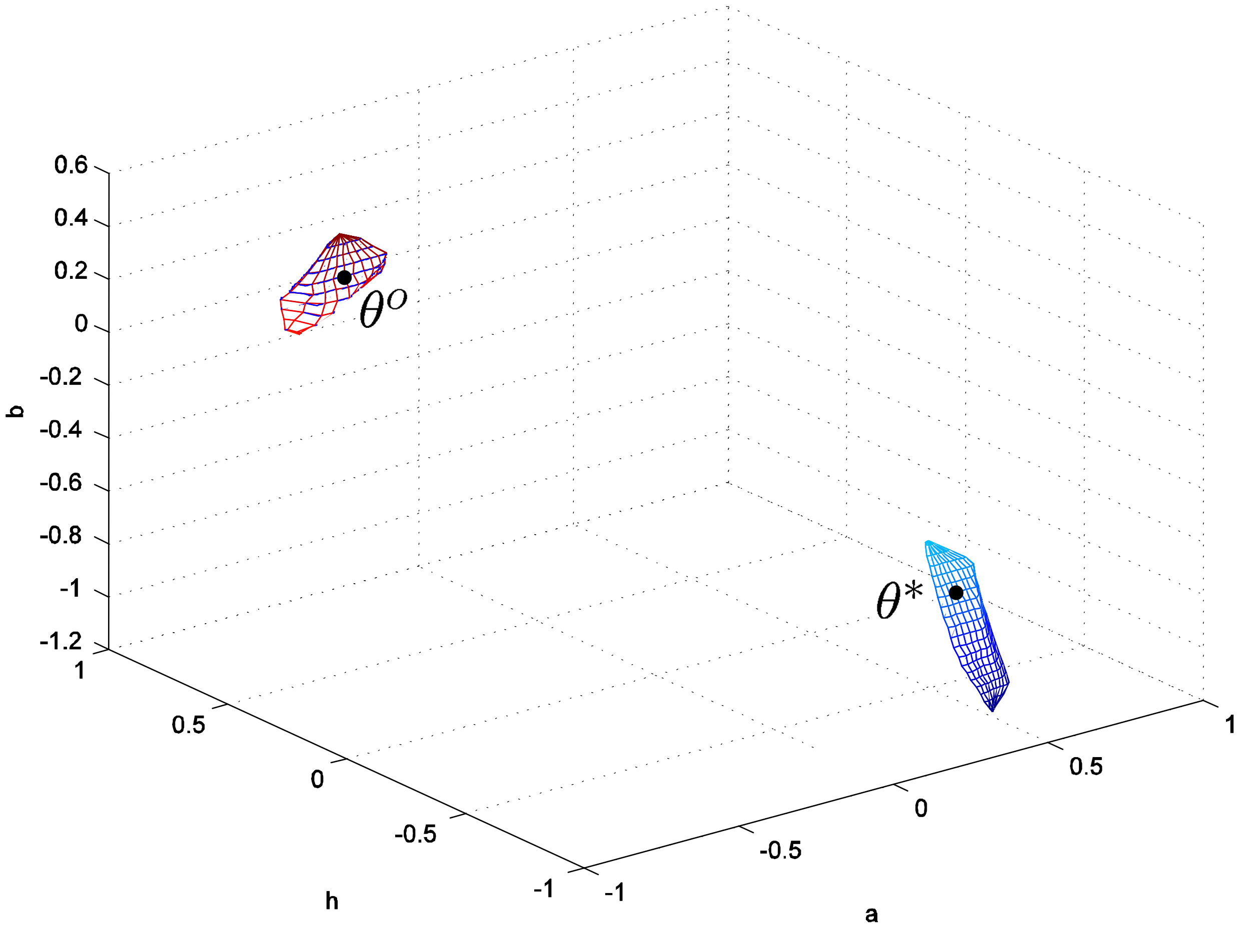


... use asymptotic theory

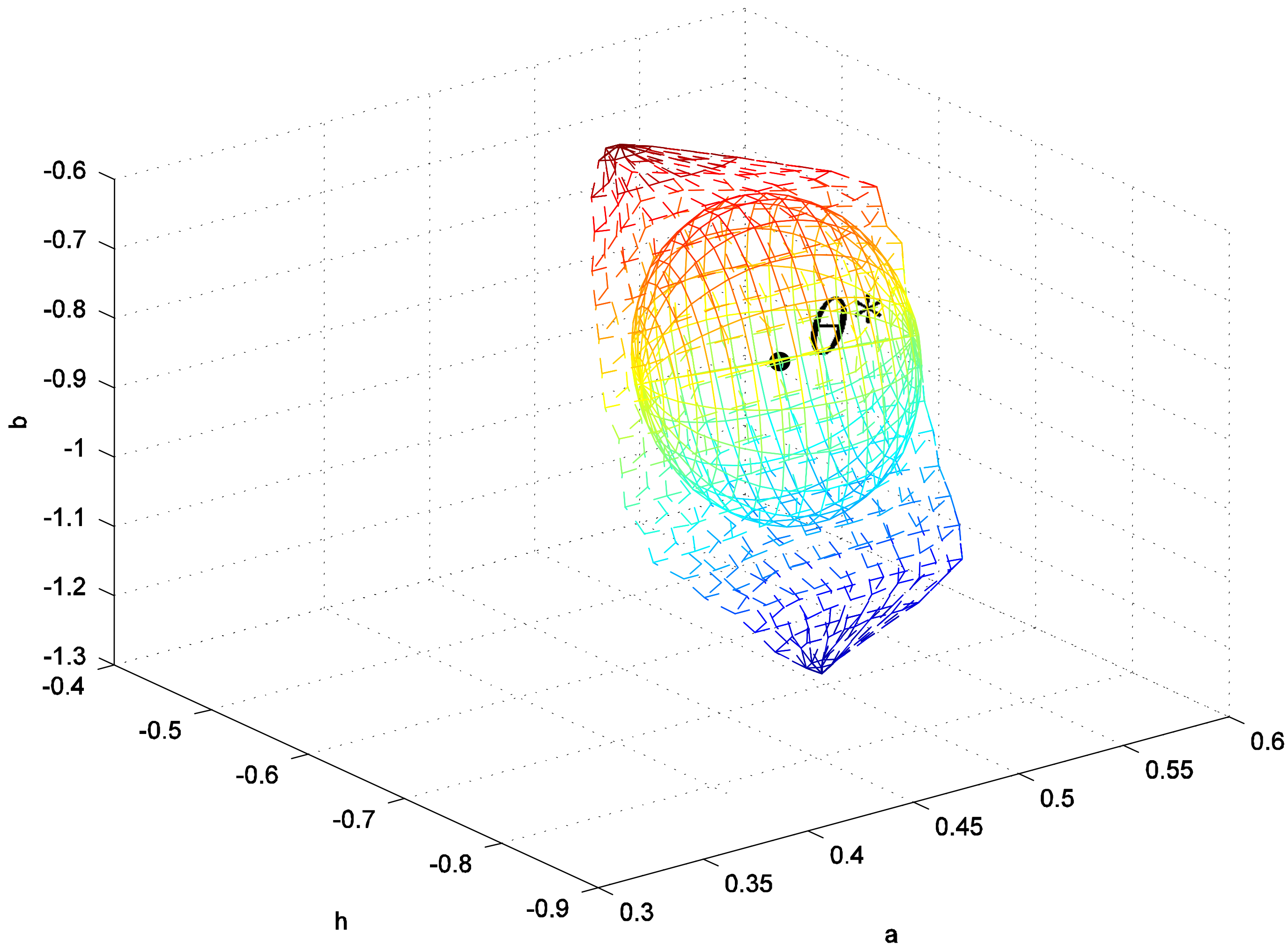


$$Pr = 90\%$$

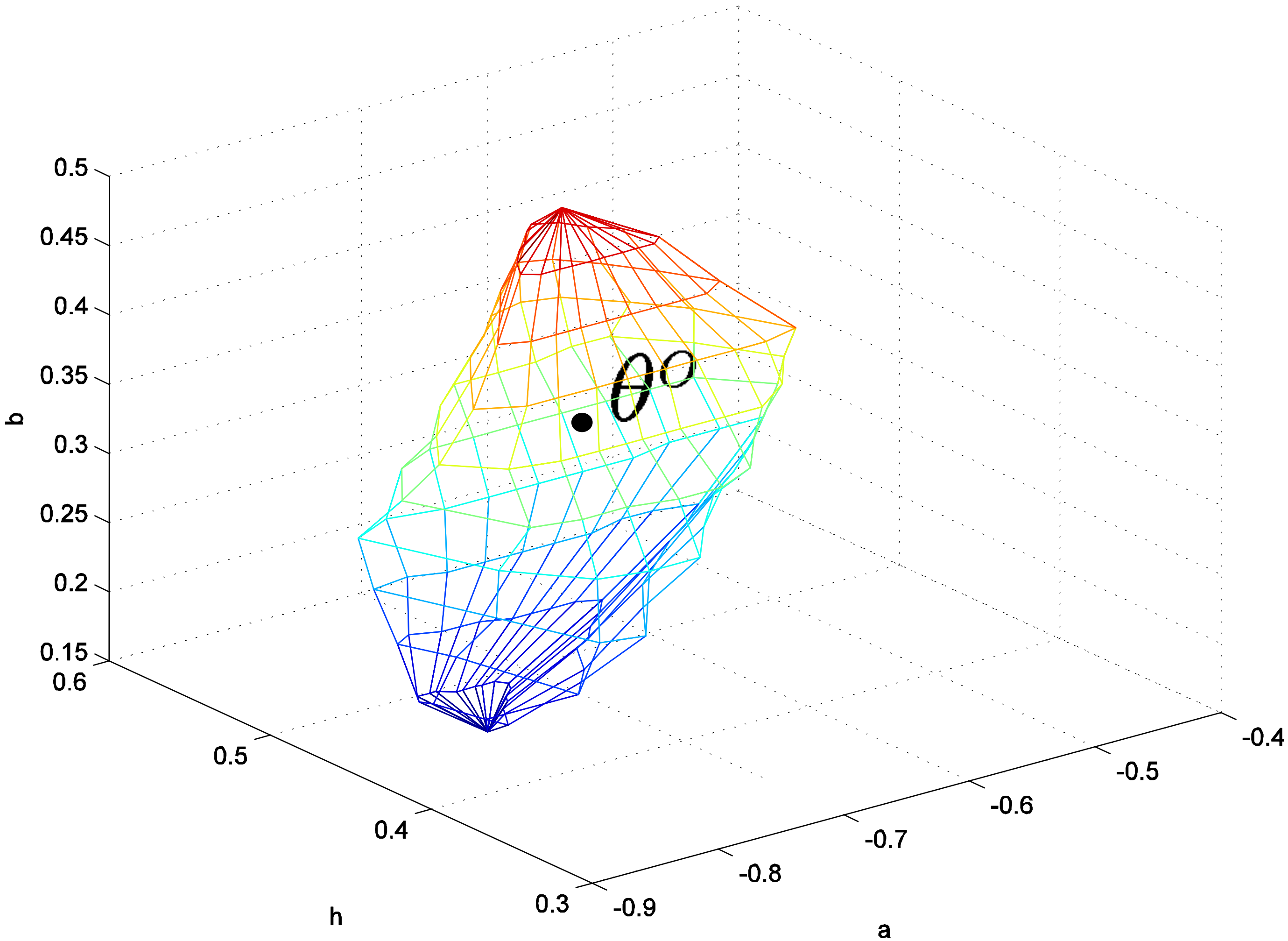
... use LSCR



$$Pr = 90\%$$

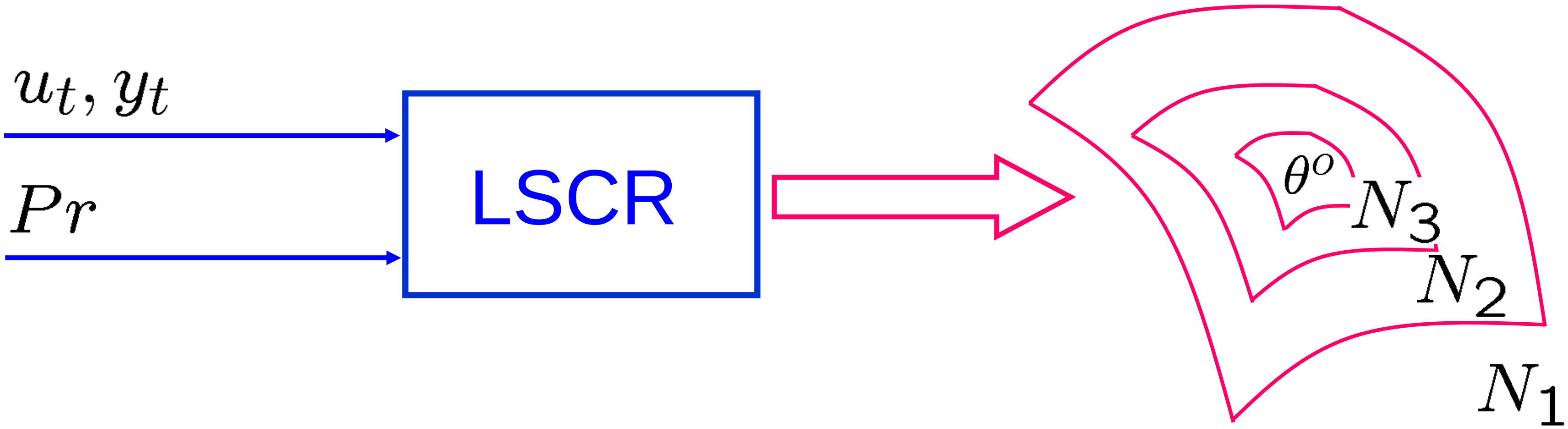


b
-0.6
-0.7
-0.8
-0.9
-1
-1.1
-1.2
-1.3
-0.4
-0.5
-0.6
-0.7
-0.8
-0.9
h
0.3
0.35
0.4
0.45
0.5
0.55
0.6
a
θ^*



b
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
h
0.6
0.5
0.4
0.3
a
-0.9
-0.8
-0.7
-0.6
-0.5
-0.4
θ^0

LSCR - properties



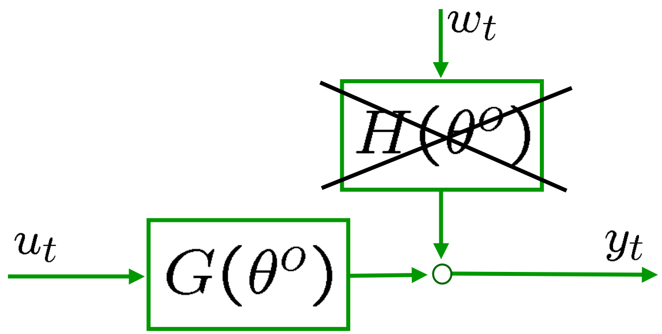
- the region shrinks around θ^o
- for any N, $\theta^o \in$ region with given Pr
  despite no assumption on level of noise is made

PART III: Extensions

- w_t non-symmetrically distributed
- nonlinear systems (bi-linear systems)

UNMODELED DYNAMICS

- estimate $G(\theta^o)$ only



- unmodelled dynamics $$y_t = b_1^o u_{t-1} + \cancel{b_2^o u_{t-2}} + w_t$$

IN CONCLUSION:

LSCR is a new framework for sys id that provides guaranteed results with minimal assumptions

and it represents an exciting topic of research