

# A Model of a Bubbling Fluidized Bed Combustor Oriented to Char Mass Estimation

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**Abstract**—Fluidized bed techniques are employed in coal combustion power plants, because they allow for the use of low-quality fuel and result in low pollutant emissions. The most important dynamical difference between fluidized beds and normal combustion chambers, is that the former are characterized by a significant loading of carbon, usually absent in conventional plants. Fluidized beds are remarkably difficult to model, since the process is characterized by a series of complex thermal and mechanical interactions. In this paper, a nonlinear lumped parameter model of a bubbling fluidized bed is presented, which captures the leading dynamics of the process. Furthermore, it is shown that this model can be successfully used to estimate the total carbon load based on an easily accessible plant variable, namely the bed temperature. Estimating the carbon load is of interest for the control of the plant.

**Index Terms**—Kalman filtering, modeling, nonlinear systems, power generation, state estimation.

## I. INTRODUCTION

THE fluidization technique is often used in processes where a nonhomogeneous reaction takes place, because of the wide contact surface between solid and gas. Known since the 1920's, fluidized bed technology was applied mostly in the chemical and petroleum industry. Only since the 1970's has it made an appearance in the power plant realm. Compared to a conventional power plant equipped with pollutant abatement systems (DENOX, DESOX), a fluidized bed combustor (FBC) has the advantages of simple operation and the capability of using low quality fuel, such as nonpulverized coal, mine residues and waste. Furthermore, the lower operating temperature of an FBC (approximately 850°C), leads to lower emissions of NO<sub>x</sub> and an improvement in the desulphurization process (the limestone present in the fluidized bed reacts with SO<sub>2</sub> to produce gypsum).

In conventional combustion chambers, the pulverized coal takes less than a second to burn out. However in an FBC, the burning time of a coal particle is remarkably higher, due primarily to the reduced temperature and larger dimension of the particles. In addition, the interactions between coal particles,

bed air bubble dynamics, and heat transfer to the steam generator are not straightforward to describe.

Therefore the modeling of an FBC requires the solution of nontrivial aspects. Usually when a spatial description of the thermodynamic state is needed, a finite element model is utilized. There also exist a large number of applications, including plant supervision and combustion control, where the estimation of global variables such as the average bed temperature or the fuel inventory (the carbon mass present in the bed), is preferred. In this case, a simplified model, based on a limited number of states, appears to be more appropriate.

When measures of some characteristic variables are available, the performance of such a simplified model can be improved by introducing suitable filtering techniques. This would entail integrating information given by physical laws with those obtained from measurements on the real plant. This approach is particularly necessary in an FBC, since the full set of state variables is not known empirically.

Direct, real-time measurement of the *char loading*, i.e., the coal mass after the release of its volatile matter and water content, can be carried out only by resorting to expensive and weakly reliable techniques, e.g., the steady-state withdrawal and analysis of a sample of bed content. On the other hand, an estimate of the char loading is necessary, since it is one of the key variables in the FBC process, describing the potential amount of heat available in the bed. For instance, an excessive rise in bed temperature and even bed smelting may occur if this variable gets too high. In the opposite case, low levels of the char content may cause difficulties in controlling the combustion power. Finally, the char loading affects the combustion efficiency and the emission of NO<sub>x</sub> [1].

The purpose of this paper is to present the results of research jointly developed by the Centro di Ricerca di Automatica (CRA) of ENEL S.p.A. (the Italian electricity board) and the Dipartimento di Elettronica e Informazione (DEI) of the Politecnico di Milano. The main objective was to develop a simple nonlinear model of an FBC, which is able to capture global dynamic effects of the plant, and integrate it with information available by temperature measurement. This will ultimately improve the performance of the FBC, and provide a reliable estimation of the char mass, without direct measurement.

This work mainly refers to a 1MW pilot bubbling FBC plant located at the ENEL research station in Livorno, put into operation in 1990. Preliminary reports on the project can be found in [2] and [3].

The model design ended up being a second order model, having the char mass and the temperature as state variables. An

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additional state variable can be introduced in the case when bed load or discharge takes place. Since the model is inherently nonlinear, the problem of state estimation was treated by means of an extended Kalman filter (EKF) algorithm.

This paper is organized as follows: after a brief presentation of the structure and operating conditions of the plant (Section II), a lumped parameters model of the Livorno FBC is described (Section III) (this model is based on the physical description of the plant, but it is simple enough to be a valid instrument for supervision, control, and real-time estimation purposes). Tuning of the model and the results obtained in state estimation are described in Sections IV and V, respectively, and some concluding remarks are reported in Section VI.

## II. DESCRIPTION OF THE PLANT

An FBC combustor contains a bed of small diameter particles such as sand. When gas is uniformly injected from the bottom with a velocity exceeding a threshold value, the minimum fluidization speed ( $u_{mf}$ ), the bed level is observed to rise. The mixture between air and solid has a homogeneous physical behavior, comparable to a liquid. During the injection process, air bubbles rise from the bottom to the top of the bed. This particular condition is known as *bubbling fluidization*. For more details about fluidization dynamics, see [4].

FBC's can be schematically represented by Fig. 1. Air is pressurized, preheated, and injected into the bed through a distributor, which is designed to produce a uniform air flow. The injected air passes through the bed, comprised of sand, limestone ( $\text{CaCO}_3$ ) and fuel (usually coal), which takes part in the chemical reactions of combustion. Thermal power is absorbed by bed heat exchangers and wall pipes. The residuals of combustion are exhaust gases blown out through the chimney. The desulphurization reaction between limestone and  $\text{SO}_2$  produces  $\text{CaSO}_4$  (discharged out the bottom of the combustor), and  $\text{CO}_2$ . The pilot FBC plant in Livorno is about 7 m high and has a circular section of approximately  $1 \text{ m}^2$ . It usually operates at  $850^\circ\text{C}$ , a temperature that has been optimized to obtain maximum efficiency in desulphurization. The feed coal flow rate is around  $110 \text{ kg/h}$  and air is injected at  $1250 \text{ kg/h}$ .

Note that the temperature of  $850^\circ\text{C}$  is typical as an operating point for FBC plants, whereas the coal flow and the air flow rate may cover wide ranges depending on the size, the generated power and the purpose of the specific combustor. Nevertheless, there is a strict relation between coal and air flow rates, to ensure clean and efficient combustion.

There are four main subprocesses which the coal particles undergo in FBC plants. When the particles are introduced into the combustor, they immediately dry up due to high temperature, so that their water content is completely released. This subprocess is called desiccation. The pressure of gases such as carbon dioxide, hydrogen, and methane present in the particles rise sharply, causing the particles to break into fragments and the gases (volatile matter) to devolatilize. At the temperature of  $850^\circ\text{C}$  and in the presence of oxygen, the carbon oxidizes to produce primarily carbon dioxide and small amounts of carbon monoxide. This is the combustion subprocess. The abrasion sub-process is due to the attrition which occurs between

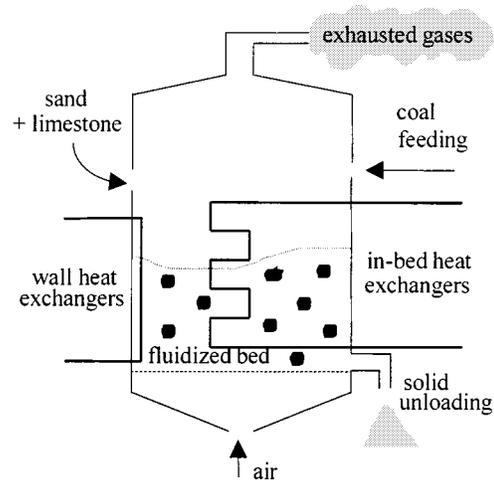


Fig. 1. The FBC combustor.

sand and coal. This leads to a decreased diameter of the coal particles, and to the production of particles of very small diameter (the *finer*) which partially burn in the bed. Finally, the coal particles react with the oxidiser in the combustion process.

A thorough model based on a physical description of the plant was developed by ENEL-CRA in [5]. It is a nonlinear model with distributed parameters, based on a finite element approach. The model can describe the operating condition of the combustor with high precision, and has been used for simulation purposes. The bed and the upper combustor (the *freeboard*) are subdivided into cells, characterized by nonuniform properties. This is mainly due to the complex geometry of the in-bed pipes changing the heat absorption in different zones. Heat absorption is modeled by also dividing the walls and in-bed pipes into cells. This model has been validated in Livorno by ENEL S.p.A. through direct plant measurements, carried out at different operating configurations, both static and dynamic [5]. In the proceeding section, this model will be referred to as the *simulator*.

## III. THE MODEL

The first objective is the development of a simple, lumped parameters model of the phenomena taking place in FBC plants. Although the simplified model is based on the same physical principles and governing law as the simulator, the design of the former comes from a completely different perspective. The result from giving a description of the heat generation in terms of global variables leads to the formulation of a uniformity hypothesis, while in the simulator the precise description of local phenomena is an issue.

As a result, it is assumed that the coal particles have a uniform diameter, and that desiccation and devolatilization dynamics are neglected. The coal and bed temperatures, uniform throughout the bed, are assumed to evolve with the same dynamics, since the char thermal capacity is negligible with respect to those of solids. Finally, the relationship between the bed-water exchange surface and bed height is assumed to be linear, neglecting the geometry of the in-bed pipes.

The model structure is modular, as shown in Fig. 2;  $w_{ci}$  (the coal flow rate)  $w_a$  (the air flow rate),  $T_a$  (the input air temper-

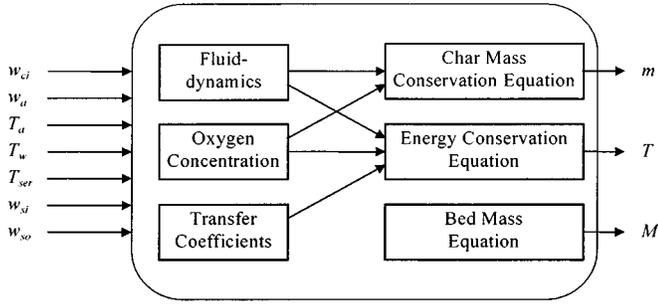


Fig. 2. The model structure.

ature),  $T_w$  (the wall pipes temperature),  $T_{ser}$  (the in-bed heat exchanger temperature),  $w_{si}$  (the input solid flow rate) and  $w_{so}$  (the output solid flow rate) are input variables, while  $m$  (the char mass),  $T$  (the average solid temperature) and  $M$  (the solid mass) are state variables. Each module of Fig. 2 is examined in turn in order to develop an adequate model.

### A. Fluid-Dynamics

A simplified description of the complex fluid-dynamics of the plant can be achieved by the two phase theory [6], according to which the bed is composed of:

- 1) a bubble phase (without solids). Denoting  $u$  as the superficial speed of the inlet air, and  $w_{ab}$  the air mass flow rate through the bubble phase, the basic relationship among these quantities is

$$w_{ab} = f_w \rho_g A_t (u - u_{mf}) \quad (1)$$

which has the same structure as standard two-phase theory except for the introduction of the correction coefficient  $f_w$  [7]. In (1),  $A_t$  is the cross section of the bed and  $\rho_g$  is the air density. The superficial velocity  $u$  is determined as follows:

$$w_a = \rho_g A_t u.$$

- 2) an emulsion phase (containing the solids) in which the air mass flow rate ( $w_{ae}$ ) is given by

$$w_{ae} = w_a - w_{ab} = \rho_g A_t ((1 - f_w)u + f_w u_{mf}).$$

The superficial gas velocity at minimum fluidization  $u_{mf}$  and the corresponding bed voidage  $\epsilon_{mf}$  are related by the Ergun equation [8], whereas an empirical correlation by Thonglimp [9] is used to obtain  $u_{mf}$ .

### B. The Char Mass Conservation Equation

Upon heating, coal releases its water content and then decomposes to produce char and volatile matter. These phenomena are characterized by a very short-time scale in comparison with the char combustion dynamics, so they can be described by algebraic equations. Precisely, the mass flow rate of volatiles,  $w_{vol}$ , is determined as a fraction,  $x_{vol}$ , of the feed coal flow rate,  $w_{ci}$ :

$$w_{vol} = x_{vol} w_{ci}.$$

The char flow rate,  $w_{ch}$ , is the balance between the coal, volatiles, and evaporating water flow rate

$$w_{ch} = (1 - x_w - x_{vol}) w_{ci}$$

where  $x_w$  is the fraction of evaporating water contained in the feed coal. The analysis of coal composition allows the determination of  $x_{vol}$  and  $x_w$ .

The combustion of char is a heterogeneous reaction between carbon and oxygen and involves the surface of the char particles only. The diameter of the char particles decreases progressively with a shrinking velocity, dependent on combustion and attrition with adjacent solid particles. The global char mass conservation equation is given by

$$\frac{d}{dt} m = w_{ch} - w_{abr} - w_{br} \quad (2)$$

where  $w_{abr}$  is the mass flow rate of char consumed by attrition, and  $w_{br}$  is the mass flow rate of burning char.

The abrasion flow rate  $w_{abr}$  is roughly proportional, through a determined constant  $k_{abr}$ , to the excess of inlet air velocity  $u$  with respect to  $u_{mf}$ , and to the particles total surface [10]. By defining  $D_{abr}$  as the average abrasion diameter, one can introduce

$$w_{abr} = k_{abr} \frac{m}{D_{abr}} (u - u_{mf}). \quad (3)$$

Burning takes place at the surface of each particle due to oxygen diffusion from the bulk of the emulsion phase. Under the spherical assumption, the surface is given by

$$S_{br} = \frac{6m}{\rho_{ch} D_{br}}$$

where  $D_{br}$  is the conventional average burning diameter, and  $\rho_{ch}$  is the char density. The burning flow rate mainly depends on the size of the coal particle, the amount of char available in the bed, and the oxygen concentration in the emulsion phase  $C_{O_2}$ , averaged along the vertical extension of the bed. This yields

$$w_{br} = \frac{12h_c}{32h_c + 12h_m} \frac{1}{1 - x_{cn}} \frac{6h_m m}{\rho_{ch} D_{br}} C_{O_2} \quad (4)$$

where  $x_{cn}$  is the fraction of unburned residues. The parameters

$$h_c = AT e^{-\frac{E_a}{RT}}$$

and

$$h_m = 2\epsilon_{mf} \frac{\delta_g}{D_{br}}$$

are the so called reaction rate [11] and mass transfer coefficients [12], which, respectively, account for limitations in combustion due to chemical kinetics and oxygen transport speed in the neighborhood of the coal particle. Here,  $E_a$  is the activation energy for the coal burning reaction,  $A$  is a constant, and  $\delta_g$  is the oxygen diffusion coefficient in the immediate surroundings of the coal particle. Note that the average diameter  $D_{br}$  in (4) may not coincide with diameter  $D_{abr}$  of (3). This is due to the diversity of the surfaces involved in the related physical phenomena. In fact, combustion takes place not only on the external

surface of the char particle, but also in some small cavities inside the particle, so that the *equivalent* surfaces of the char particle related to combustion and attrition may differ. Typically  $D_{abr} < D_{br}$ .

### C. Fines Combustion

The fines combustion rate in the bed is controlled by the availability of oxygen in the emulsion phase and by the so called *percolation velocity* of fines along the bed [5]. Since the fines combustion phenomenon is extremely complex, some simplification is required to develop a tractable model. We have assumed that only a fraction,  $\theta$ , of the fines mass flow rate,  $w_{abr}$ , produced in the bed is actually burned:

$$w_{brf} = \theta w_{abr}$$

where  $\theta$  is given by the empirical correlation

$$\theta = \frac{M_{fin}C_{O_2}}{1 + M_{fin}C_{O_2}}.$$

The coefficient  $M_{fin}$  will be estimated based on the reference model of the plant.

### D. The Energy Conservation Equation

The energy balance takes the following form:

$$(c_s M_s + c_{ch} m) \frac{d}{dt} T = P_{br} + P_{vol} + P_{brf} - (P_{ser} + P_w + P_a) \quad (5)$$

where  $c_{ch} m$  is the char heat capacity and  $c_s M_s$  the solid heat capacity.

The input power is the sum of the heat per unit time released by the burning char ( $P_{br}$ ), the burning fines ( $P_{brf}$ ) and burning volatile gases ( $P_{vol}$ ), while the output power is absorbed by the bed ( $P_{ser}$ ), wall ( $P_w$ ) heat exchangers, and transported by air flowing through the bed ( $P_a$ ). In (5), some terms have been neglected. Namely, the power output associated to discharged solids, since in normal operation  $w_{si}$  and  $w_{so}$  are very small, and the power output needed for to heat the feed coal is a small fraction of the total heat.

The thermal power released by char combustion,  $p_{br}$ , is expressed as

$$P_{br} = w_{br} H_{ch}$$

where  $w_{br}$  is the char burning rate, according to (4) and  $H_{ch}$  is the combustion heat.

The energy rate released by volatile matter combustion is proportional to the mass flow of volatiles, where  $H_{vol}$  is the heat of combustion of volatiles

$$P_{vol} = w_{ci} x_{vol} H_{vol}.$$

The combustion of fines (whose combustion heat is  $H_{ch}$ ) is represented by  $P_{brf}$ , which is given by

$$P_{brf} = w_{brf} H_{ch}.$$

The term  $P_a$  is the power delivered to heat the air mass flow from the inlet temperature (enthalpy  $h_{ai}$ ) to the bed temperature

(air enthalpy at atmospheric pressure,  $h(T)$ , is computable from appropriate tables).

$$P_a = w_a h(T) - w_a h_{ai}.$$

Finally,  $P_w + P_{ser}$  is the thermal power transferred to the cooling surfaces

$$P_w + P_{ser} = \gamma_w S_w (T - T_w) + \gamma_{ser} (T - T_{ser}).$$

$S_w$  and  $S_{ser}$  are the exchange surfaces of walls and pipes, dependent on the bed height;  $\gamma_w$  and  $\gamma_{ser}$  are the heat transfer coefficients

$$\gamma_w = F_w \frac{1}{\frac{1}{(\gamma_{rad} + \gamma_{cv})_w} + \frac{1}{\gamma_{ref}}}$$

$$\gamma_{ser} = F_{ser} [(\gamma_{rad} + \gamma_{cv})_{hor} + (\gamma_{rad} + \gamma_{cv})_{ver}]$$

where  $\gamma_{rad}$  and  $\gamma_{cv}$  are heat radiation and convection coefficients related to walls (subscript  $w$ ), horizontal (subscript  $hor$ ) and vertical (subscript  $ver$ ) pipes.  $\gamma_{ref}$  is the heat transfer coefficient of the refractory material protecting the wall pipes. All these coefficients are functions of the state variables and plant characteristics, omitted here for brevity. Moreover,  $F_w$  and  $F_{ser}$  are two empirical coefficient, taking into account uncertainties that the model cannot interpret, such as nonuniformities in bed heat absorption.

### E. The Oxygen Concentration

In the proposed model for char combustion the char reaction rate is determined by the average oxygen concentration in the emulsion phase, where all heterogeneous reactions are assumed to occur. Oxygen concentration varies vertically in both the emulsion and bubble phase due to the reactions that take place along the air flow through the bed. Furthermore, oxygen is transferred from the bubble to the emulsion phase, with a mass transfer coefficient given by a suitable empirical law [13]. Denoting  $C_{O_2e}$  and  $C_{O_2b}$  as the oxygen concentration in the emulsion and bubble phases respectively, and  $k_1$ ,  $k_2$  and  $k_{tr}$  as some given coefficients, the oxygen mass concentration equations in the emulsion and in the bubble phase are

$$\begin{cases} A_t \frac{\partial C_{O_2b}}{\partial t} + \frac{w_{ab}}{\rho_g} \frac{\partial C_{O_2b}}{\partial z} = -k_{tr} \Delta C_{O_2be} \\ A_t \frac{\partial C_{O_2e}}{\partial t} + \frac{w_{ae}}{\rho_g} \frac{\partial C_{O_2e}}{\partial z} = -k_1 C_{O_2e} + k_2 + K_{tr} \Delta C_{O_2be}. \end{cases} \quad (6)$$

The storage terms containing the time derivatives in this equation will henceforth be neglected. This is because the associated transient phenomena are very fast compared to others described in the model. Terms containing partial derivatives with respect to the vertical coordinate  $z$  are transport terms. The quantity  $k_{tr} \Delta C_{O_2be} = k_{tr} (C_{O_2b} - C_{O_2e})$  is the oxygen mass transfer between the two phases;  $k_1 C_{O_2e}$  represents the oxygen consumption rate due to char combustion, whereas  $k_2$  represents the oxygen consumption rate due to volatiles and fines combustion.

Equations (6) have been integrated with the boundary condition that at  $z = 0$ , the oxygen concentration in the bed (both emulsion and bubble phase), is equal to the oxygen concentration in the inlet air. Averaging the oxygen concentration yielded by the numerical solution of system (6) along the emulsion

phase, we can obtain the mean oxygen concentration in the emulsion phase,  $C_{O_2}$ , used in the model.

#### F. The Bed Mass Conservation Equation

Ignoring the consumption of limestone in the desulphurization reaction, we find that the total mass of sand and limestone present in the bed depends on the inlet and outlet solid mass flow rates, according to the following equation:

$$\frac{d}{dt}M_s = w_{si} - w_{so}. \quad (7)$$

#### G. The Model of the Plant

In conclusion, the equations governing the system dynamic behavior are the char mass conservation equation (2), the bed energy conservation equation (5) and the bed mass conservation equation (7)

$$\begin{aligned} \frac{d}{dt}m &= w_{ch} - w_{abr} - w_{br}, \quad (2) \\ (c_s M_s + c_{ch} m) \frac{d}{dt}T &= P_{br} + P_{vol} + P_{brf} \\ &\quad - (P_{ser} + P_w + P_a), \quad (5) \\ \frac{d}{dt}M_s &= w_{si} - w_{so}. \quad (7) \end{aligned}$$

Thus, the model of the plant is a third-order nonlinear dynamic system with  $m$  (char mass),  $T$  (bed temperature) and  $M_s$  (bed solid mass) as state variables,  $w_{ci}$  (feed coal mass flow rate),  $w_a$  (air mass flow rate),  $w_{si}$  and  $w_{so}$  (inlet and outlet solid mass flow rates) as input variables and  $T$  (bed temperature) as the output variable. The model can be further simplified by making reference to those operating conditions where the input and output solid flow rate coincide ( $w_{si} = w_{so}$ ), so that (7) can be dropped.

#### IV. PARAMETER SELECTION AND MODEL VALIDATION

The model equations contain a large number of parameters whose precise values are unknown *a priori*. These parameters are the average burning and abrasion diameters,  $D_{abr}$  and  $D_{br}$ , the constant  $M_{fin}$ , present in the modeling of the fines burning mass flow rate and finally  $F_w$  and  $F_{ser}$ , parameters appearing in the description of wall and bed heat exchange rates. Tuning of these parameters was carried out in order to match the steady-state behavior of the simplified model to that of the simulator. A typical steady-state condition (referred to as  $S1$ ), with input values are shown in Table I, was considered.

The conditions of  $S1$  were chosen according to realistic operating conditions of the Livorno plant. The tuning process first required a sensitivity analysis of the system to the parameters, followed by several steady-state simulations on the simplified model, searching for the parameter set which minimized the relative error between model and simulator. Specifically, tuning was performed using considerations that some parameters have a stronger influence on particular variables. For instance,  $F_w$  and  $F_{ser}$  influence power output. Their values were determined so that the modeled  $P_w$  and  $P_{ser}$  were close to the simulated ones. In the same way,  $D_{br}$  and  $M_{fin}$  are strictly related to power input.  $D_{abr}$  can be tuned in order to reduce the error be-

TABLE I  
STEADY-STATE CONDITION  
 $S1$

$w_{ci} = 0.12\text{kg/s}$	$T_w = 333\text{K}$
$w_a = 1.3\text{kg/s}$	$T_{ser} = 368\text{K}$
$T_a = 400\text{K}$	$M_s = 800\text{kg}$

TABLE II  
PARAMETERS VALUE

$D_{abr} = 0.00225$	$M_{fin} = 230$
$D_{br} = 0.005562$	$F_w = 0.805$
$F_{ser} = 0.5526$	

TABLE III  
MATCHING BETWEEN MODEL AND SIMULATOR AT  $S1$

	Simulator	Model	% Error
$m$ (kg)	107.30	107.32	0.01%
$T$ (K)	1367.00	1367.01	0.00%
$u_{mf}$ (m/s)	0.1015	0.1007	0.8%
$P_{br}$ (W)	$8.972 \cdot 10^5$	$9.021 \cdot 10^5$	0.55%
$P_{brf}$ (W)	$1.085 \cdot 10^6$	$1.066 \cdot 10^6$	1.67%
$P_w$ (W)	$2.422 \cdot 10^5$	$2.425 \cdot 10^5$	0.14%
$P_{ser}$ (W)	$9.588 \cdot 10^5$	$9.084 \cdot 10^5$	5.26%

tween modeled and simulated char mass. Of course, these links between parameters and system variables are not independent of each other, and these considerations can only be viewed as guidelines for the tuning process.

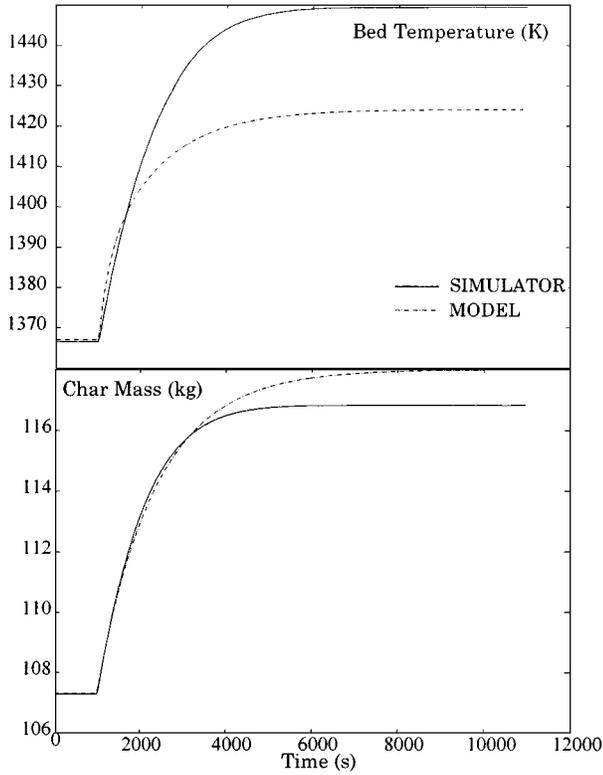
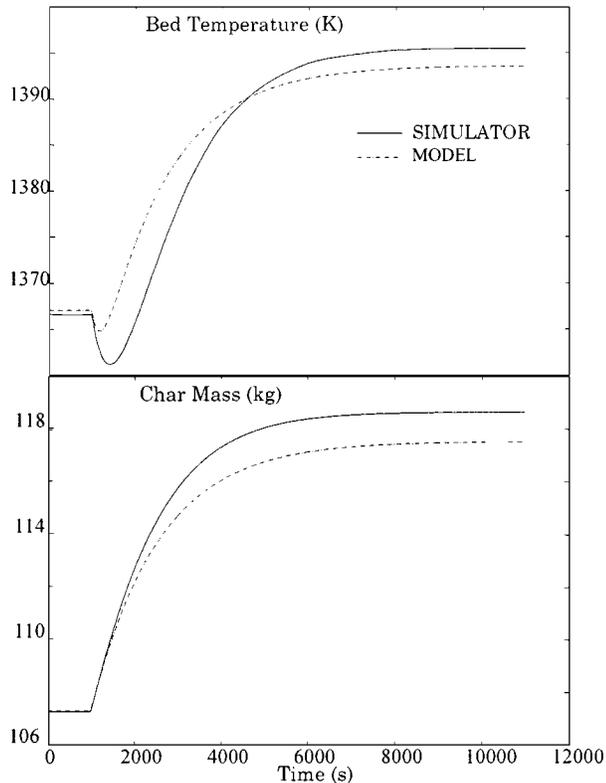
The selected set of tuned parameters is given in Table II, while in Table III a comparison among the steady-state values of the variables at  $S1$  is displayed.

We can observe that the model correctly describes the behavior of the plant, at least at  $S1$ . With the goal of validating the model with the chosen set of parameters, several tests in different steady and transient conditions were performed, as indicated in the following steps.

- 1) Starting with  $S1$ , the model was tested by matching state variable transients of the simulator and the model when a 5% or 10% step was imposed to the input variables  $w_a$  or  $w_{ci}$ . These tests showed that in a neighborhood of the  $S1$  condition, the dynamic behavior of the simplified model and simulator are quite similar [See Figs. 3(a) and 3(b)].
- 2) Two further steady-state conditions, named  $S2$  and  $S3$ , were considered (Table IV).  $S2$  and  $S3$  were obtained by modifying  $w_{ci}$ ,  $M_s$ , and  $w_a$  from the values in  $S1$ . The model behavior was tested in the neighborhoods of the additional operating points.

The comparison between the simulator and the model tuned for  $S1$  was now performed. The disagreement is now not negligible, about 5% for the char mass and for the bed temperature, as shown in Table V.

This was confirmed by transient tests starting from the new conditions (see Fig. 4). From the previous discussion, it follows that the model is sensitive to the specific operating condition; as such, it should be tuned accordingly. A set of values for the parameters is needed for each steady-state condition. There is clearly a link between the parameters and steady-state conditions.

Fig. 3(a). Negative step of 10% of  $w_a$  starting from  $S1$ .Fig. 3(b). Positive step of 10% of  $w_{ci}$  starting from  $S1$ .

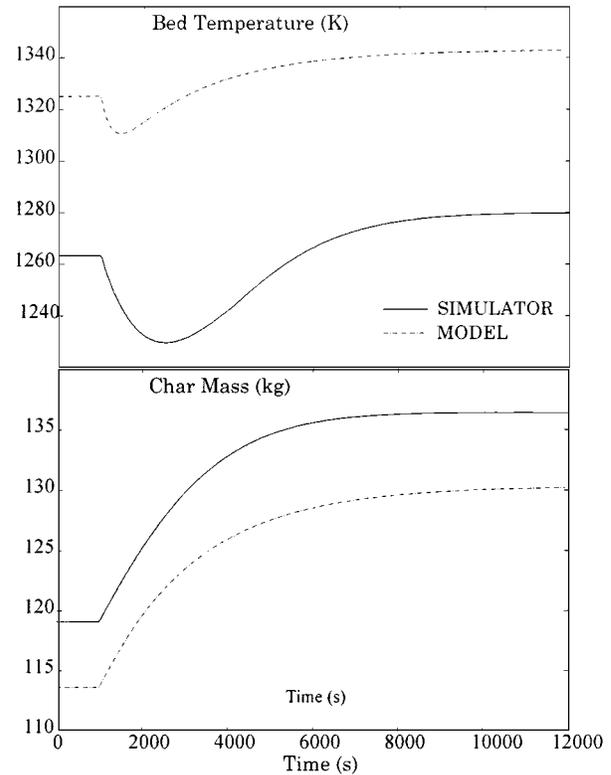
To achieve good matching between the simplified model and the simulator, it was therefore necessary to tune the model

TABLE IV  
STEADY STATE CONDITION  $S2$  and  $S3$

$w_{ci} = 0.1\text{kg/s}$	$M = \begin{cases} 800\text{kg} & \text{for } S2 \\ 1300\text{kg} & \text{for } S3 \end{cases}$
$w_a = 1.0\text{kg/s}$	
$T_a = 400\text{K}$	
$T_w = 333\text{K}$	
$T_{ser} = 368\text{K}$	

TABLE V  
MATCHING BETWEEN MODEL AND SIMULATOR AT  $S2$

	Simulator	Model	% Error
$m$ (kg)	119.1	113.6	4.62%
$T$ (K)	1263	1325	4.91%
$u_{mf}$ (m/s)	0.1062	0.1015	4.42%
$P_{br}$ (W)	$8.139 \cdot 10^5$	$8.131 \cdot 10^5$	0.10%
$P_{brf}$ (W)	$6.969 \cdot 10^5$	$8.229 \cdot 10^5$	18.09%
$P_w$ (W)	$1.765 \cdot 10^5$	$2.098 \cdot 10^5$	18.84%
$P_{ser}$ (W)	$8.600 \cdot 10^5$	$9.039 \cdot 10^5$	5.10%

Fig. 4. Negative step of 10% of  $w_a$ , starting from  $S2$ .

around  $S2$  and  $S3$ . In Table VI we report the parameter values relating to the tuned model in  $S2$  and  $S3$ .

Note that the state variables are sensitive to small variations of  $D_{abr}$  and  $D_{br}$ . The results obtained from a sensitivity analysis of the model in the neighborhood of  $S1$  show, for instance, that a 10% increase of  $D_{br}$  leads to an excess of more than 8% of the char mass. It is clear that an extremely precise tuning of the model is required around each operating point.

The need to tune the model over a range of operating conditions could be avoided by determining the parameter values as functions of the system state. However, due to the large number of parameters appearing in the model equations, this seems to be a remarkably complex task. Despite these conclusions, the

TABLE VI  
PARAMETERS VALUES

Parameters	S1	S2	S3
$D_{abr}$	0.00225	0.002255	0.002245
$D_{br}$	0.005562	0.00576	0.005715
$M_{fin}$	230	62	50
$F_w$	0.805	0.765	0.737
$F_{ser}$	0.5526	0.6532	0.6815

simplified model presented in this paper can correctly describe the main dynamics of an FBC with two or three simple lumped parameters equations, and is more tractable than the simulator model.

## V. IMPROVING THE STATE ESTIMATION

In order to improve the estimation of state variables and overcoming the problem of retuning the model according to operating condition modifications, a natural strategy would be to integrate the information provided by the model with data from available experimental measurements. Extended Kalman filter techniques (for a description of EKF see, e.g., [14]) are employed when state estimation of nonlinear systems is required; in particular, we will show how the use of an EKF applied to the simplified model for S1 leads to fair state estimations, even under different operating conditions.

We assume that the plant operates with a constant mass of solids, so that the model considered is second order. The objective of the filter is to estimate the state variable  $m$ , starting with the real time measurement of the temperature  $T$ .

The state, output, and input variables are given by

$$x = \begin{bmatrix} m \\ T \end{bmatrix}, \quad y = T, \quad u = \begin{bmatrix} w_{ci} \\ w_a \end{bmatrix}.$$

To apply EKF concepts, the plant model described by (2) and (5) is modified to incorporate disturbance terms as follows:

$$\begin{aligned} \frac{d}{dt}m &= w_{ch} - w_{abr} - w_{br} + v_1 \\ \frac{d}{dt}T &= \frac{1}{c_s M_s + c_{ch} m} [P_{br} + P_{vol} + P_{brf} \\ &\quad - (P_{ser} + P_w + P_a)] + v_2. \end{aligned}$$

The disturbance vector  $v(t) = [v_1(t) v_2(t)]'$  summarizes all random effects as well as the mismatch of the simplified model from reality. Its  $2 \times 2$  intensity matrix will be denoted by  $Q$ .

$$Q = \begin{bmatrix} q_{11} & q_{12} \\ q_{12} & q_{22} \end{bmatrix}.$$

Analogously, the output equation

$$y = T$$

is modified to

$$y = T + w$$

with the white noise,  $w(t)$ , having variance  $R$ , accounting for measurement errors.

For application of the EKF, a linearized model of the plant is required. The dynamic and output matrices of such a linearized model, as defined in [14], are obtained by numerically calculating the derivatives of (2) and (5). This method is preferred over analytically evaluating the derivative, mainly for its flexibility. In fact, if the model is to be corrected or modified, the adoption of a numerical method avoids analytical recalculation of the derivatives. Finally, since the filter design requires the solution of a Riccati equation based on the linearized model together with  $Q$  and  $R$ , starting from some initial condition—say  $P_0$ , these last three matrices must be properly assigned. This is the most delicate phase in the design of the estimation algorithm.

$R$  describes the intensity of the measurement error from the temperature transducer: a larger  $R$  implies less confidence in the measurements. By considering experimental data obtained from tests at the Livorno plant, the value of  $R = 10$  was taken. The initial value of the estimation error variance,  $P_0$ , affects the convergence speed, and therefore can be seen as a reliability index of the *a priori* estimated initial state. The intensity matrix  $Q$  is the only parameter whose tuning attempts to optimize the steady-state estimation, since  $R$  depends on the instrument precision and  $P_0$  uniquely affects the convergence speed of the algorithm. The relative weights between  $R$  and  $Q$  play an important role in the filter tuning process [15]. If the  $Q$  entries are higher than  $R$ , the filter will have confidence in the measurement set; in the opposite case, the measurements are not considered to be reliable data.

The procedure for the selection of  $Q$  in the case of the char mass estimation problem is based on the possibility of attaining direct or indirect information on the real value assumed by the state variable. In general, the rationale is to choose  $Q$  so that, at any time  $t$ , the error between the estimated char mass and the true value is minimized. If we assume that the true value of the char mass  $\bar{m}_i$  is available in  $N$  steady-state conditions, an appropriate measure of the performance of the EKF is given by the cost function

$$J = \frac{1}{N} \sum_{i=1}^N |\bar{m}_i - \hat{\bar{m}}_i|$$

where  $\hat{\bar{m}}$  is the estimated char mass in the  $i$ th steady-state condition for a given  $Q$ .  $Q$  can then be selected over a feasible range to minimize  $J(Q)$ .

Unfortunately, this method was not directly applicable, since the few char mass experimental measurements carried out by ENEL, using expensive sampling withdrawal techniques, had very low reliability and were affected by non systematic errors. Measurement unreliability is due to the nonuniform char distribution. The extracted 1-kg bed sample may have a measured char density different from the mean value. For these reasons, in the tuning of matrix  $Q$ , we resorted to the simulator in place of the real plant.

By denoting with the value of the char mass as  $\bar{m}_{si}$  in the  $i$ th steady-state condition provided by the simulator, the cost function becomes

$$J' = \frac{1}{N} \sum_{i=1}^N |\bar{m}_{si} - \hat{\bar{m}}_i|.$$

TABLE VII  
OPTIMUM VALUE FOR  $J'$

		$q_{22}$		
		0.01	<b>0.1</b>	1
$q_{11}$	$10^{-5}$	1.7804	<b>0.6937</b>	1.0230
	$10^{-4}$	4.8514	1.8109	0.6966
	$10^{-3}$	6.3898	4.9312	1.8195

In Table VII,  $J'$  is evaluated and minimized considering five steady-state conditions around  $S1$ , with a grid of nine values of  $Q$ , assumed to be diagonal. The chosen steady-states were  $S1$  and those obtained by a 10% increase or decrease in the amount of air and coal flow rate, respectively.

The chosen value of  $Q$  corresponding to the minimum  $J'$  was

$$Q = \begin{bmatrix} 10^{-5} & 0 \\ 0 & 10^{-1} \end{bmatrix}.$$

The estimation algorithm was tested in steady-state and transient conditions, repeating the tests presented in Section IV with the EKF tuned as previously shown. The real measurement of the temperature was replaced with the simulator output variable. An artificial Gaussian white noise of variance  $R$  was added in order to account for measurement disturbances. In these tests the value

$$P_0 = \begin{bmatrix} 10^{-3} & 0 \\ 0 & 10^{-3} \end{bmatrix}$$

was selected. The choice of small values for the  $P_0$  entries is a consequence of the fact that the simulations are carried out with an initial filtered state close to the simulated one. It has been observed that larger values lead to a slower convergence speed, even if filtered and simulated initial states are different.

- 1) Starting from  $S1$ , new transients were calculated when a 5% or 10% step of the inputs was imposed. Figs. 5(a)–5(b) show the matching between simulator, model, and EKF for a specific transient. When the model is well tuned about a steady condition, one can observe that the filter and model have similar behavior in the char mass estimate. In this case, the information carried by measurements is almost useless.
- 2) In the  $S2$  case, the model tuned to  $S1$  poorly fits the simulator data (see Section IV). In contrast, application of the filter leads to an improvement in the mass estimation. From Fig. 6, one can see that the estimate, starting from the  $S2$  condition, is closer to the simulator value.

Several different transient tests starting from the  $S2$  operating point with step inputs of  $w_{ci}$  or  $w_a$  were performed. They all showed that the Kalman filter-based algorithm with  $Q$  tuned as described above, allows one to obtain an adequate estimate of the char mass. A set of these simulations is reported in Figs. 7(a), 7(b), and 7(c), where one can observe the improvement in char mass estimation provided by the EKF. By evaluating the differences between the final and the initial steady-state related to each step response, the percent EKF estimation error is always less than the open-loop model. Surprisingly good results are shown in Figs. 7(a) and 7(b), where the error is reduced from 30–40% to 1–5%. In Fig. 7(c) the estimation error drops from 36% to 27%, but if the absolute

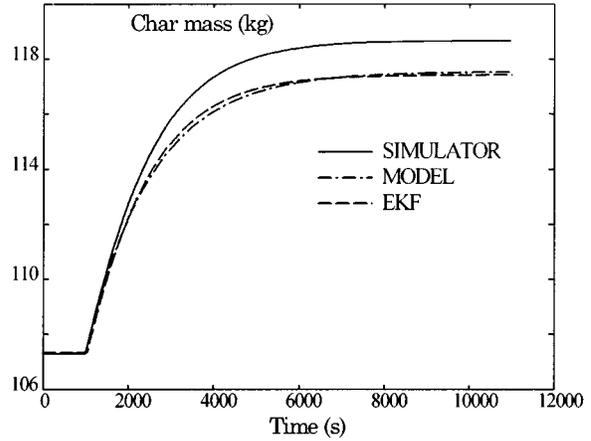


Fig. 5(a). Negative step of 10% of  $w_a$  starting from  $S1$ .

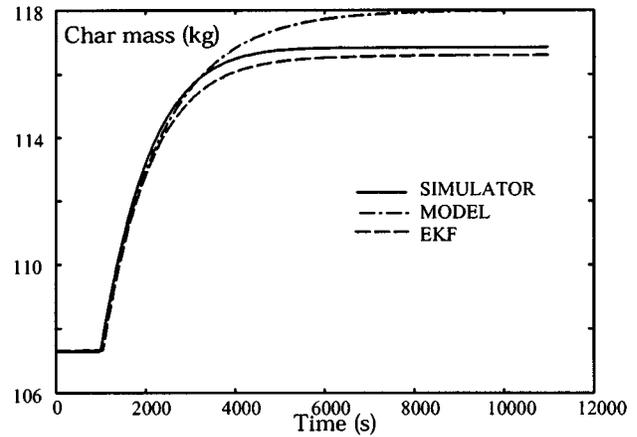


Fig. 5(b). Positive step of 10% of  $w_{ci}$  starting from  $S1$ .

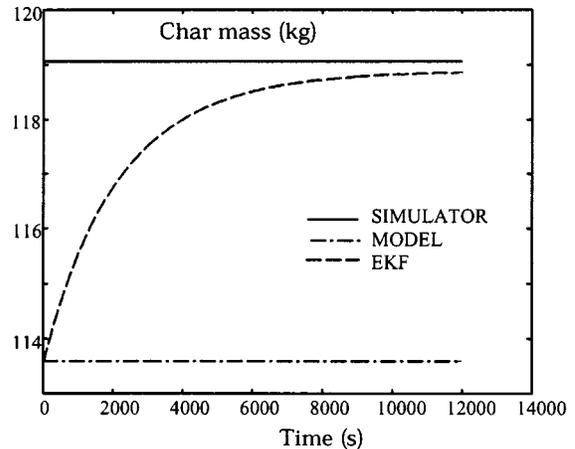


Fig. 6.  $S2$ : convergence of the estimate to the simulated char mass.

value is considered, the char mass estimation error is just few kilograms.

## VI. CONCLUDING REMARKS

In this paper, the main features of a simplified model of a FBC plant are presented. This is a lumped parameter third-order model, and contains five uncertain parameters that must be

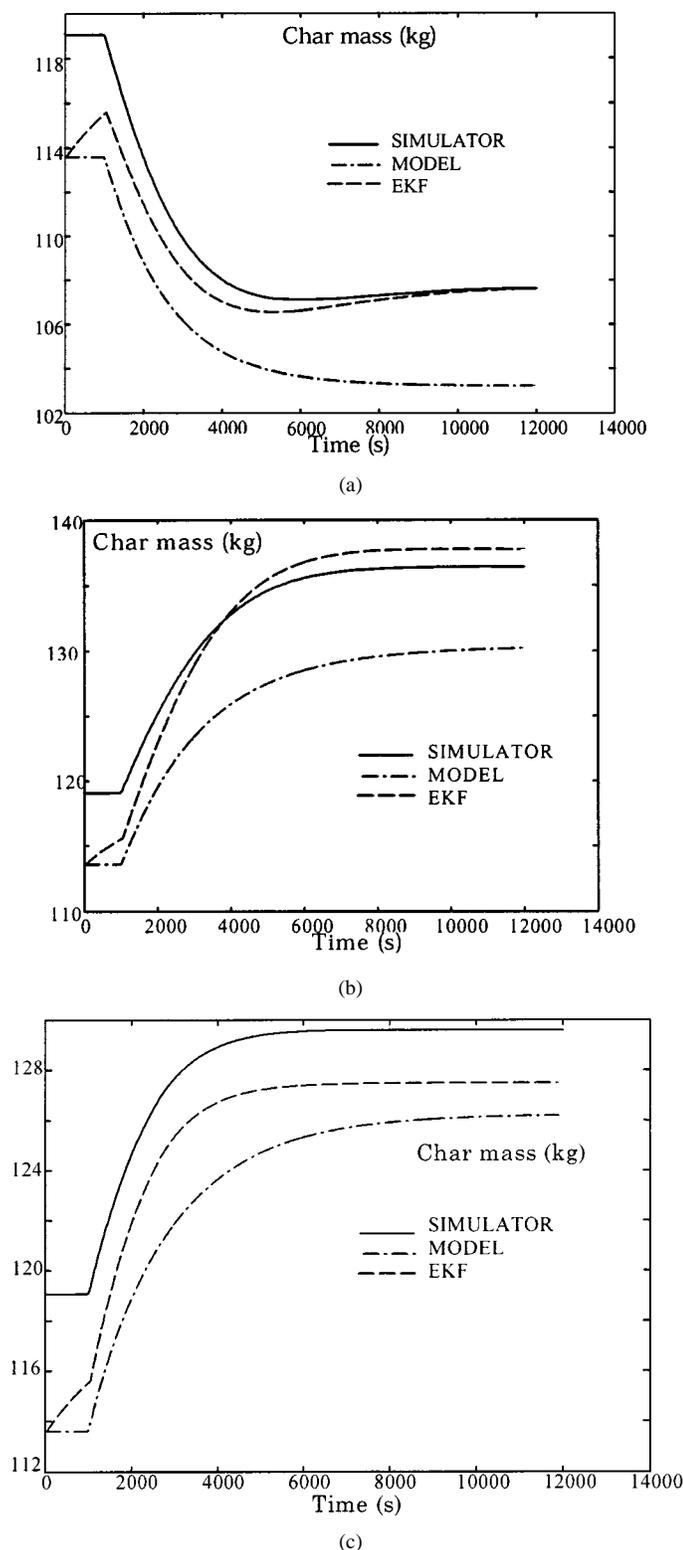


Fig. 7. (a) Positive step of 10% of  $w_{ci}$  starting from  $S_2$ . (b) Negative step of 10% of  $w_{ci}$  starting from  $S_2$ . (c) Positive step of 10% of  $w_{ci}$  starting from  $S_2$ .

tuned. The design of such a model arises from the need to capture the main process dynamics, in order to support supervision and control strategies in the combustion chamber, and to provide an estimate of important unknown state variables, such as the char mass.

In general, it has been shown that the simplified model has good performance only in the neighborhood of the steady-state condition where the parameters have been tuned. A significant improvement of the state estimation is achieved by utilizing EKF techniques employing temperature measurements.

To apply the estimation algorithm, one must select the matrices  $Q$ ,  $R$  and  $P_0$ . Guidelines to design a suitable  $Q$ , based on simulations, have been provided. Furthermore, the application of the algorithm allows one to avoid tuning the model parameters every time there is a change in the plant operating point.

We can argue that this method, developed for the Livorno plant, can be applied in similar FBC plants, after having returned the main parameters of the model and the EKF to obtain good performance.

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