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A Penalized Identification Criterion for Securing Controllability in Adaptive Control*

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Abstract

Reportedly, standard identification algorithms do not guarantee the controllability of the estimated system. In this paper, a penalized least squares (PLS) identification criterion is proposed to overcome this difficulty. The criterion is shown to provide estimated systems which exhibit an uniform controllability property through time. Moreover, the Lai and Wei upper bound for the least squares estimation error ([1], Theorem 1) is still valid for PLS. This ensures a safe use of the proposed method in adaptive control applications. The effectiveness of the method is illustrated by a general adaptive stability result valid for PLS-based certainty-equivalent adaptive control schemes.

Key words: uniform controllability, adaptive control, least squares identification, stochastic systems, penalized identification

AMS Subject Classifications: 93C40

1 Introduction

A classical technical trap encountered in the subject of adaptive control of nonminimum phase systems is the possible occurrence of pole-zero cancellations in the estimated model transfer function (see *e.g.* [2]-[9]). As a matter of fact, standard identification methods do not guarantee that the controllability property of the original system (*i.e.* the coprimeness of numerator and denominator in the system transfer function) is preserved when the signals generated by the control system fail to provide sufficient excitation, as it is often the case under closed-loop operating conditions. In turn, the lack of controllability in the estimated system leads to a paralysis in the adaptive control law selection.

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Basically, two different approaches have been proposed in the literature to cope with the controllability problem. One consists in modifying the identification algorithm in order to force the parameter estimates to tend to a preassigned convex region to which the true parameter belongs and such that all the models in that region are controllable ([7] and [8]). Unfortunately, the required a-priori knowledge on the system parameter represents a restrictive assumption and it confines the use of this approach to the cases in which the parameter uncertainty is highly structured. The second approach ([2]-[6]) secures the estimated model controllability by redressing the parameter estimate before using it to compute the control law. Precisely, an extra term is added to the least squares estimate which depends on the least squares covariance matrix in such a way that the closed-loop identification properties of the least squares algorithm (*i.e.* the ability of identifying the system dynamics excited in closed-loop) are preserved while securing the uniform controllability of the identified system. In this approach, no a-priori knowledge on the region to which the true parameter belongs is assumed. The explicit method for the estimate modification given in [3], however, involves a computational effort which highly increases with the order of the system.

In the present paper, we propose a solution to the long-standing pole-zero cancellation issue based on an appropriate modification of the standard least squares identification index, which does not require any assumption on the true parameter value. The new index suitably incorporates a penalization term for the parameterizations which correspond to uncontrollable—or nearly uncontrollable—models.

The major point is that the minimization of the new identification index cannot be accompanied by the obnoxious side effect of controllability violation, since uniform controllability is guaranteed. Even more so, the closed-loop properties of the least squares algorithm stated in [1], Theorem 1, are preserved. This is of crucial importance in adaptive control applications for proving stability and optimality results (see *e.g.* [10], [11] and [12]).

The dark side of the coin is that the minimization issue is not as straightforward as it is in the least squares algorithm and one has to resort to some iterative optimization procedure. This can be a problem since this minimization has to be performed on-line and it may be therefore subject to strict time limitations. Designing fast (and, possibly, approximate) minimization procedures is a challenging problem, open to further research.

2 The Penalized Least Squares Identification Method

2.1 The system

We consider a discrete time stochastic SISO system governed by the equation

$$A(\vartheta^\circ; q^{-1})\, y_t = B(\vartheta^\circ; q^{-1})\, u_{t-d} + n_t, \qquad d \geq 1 \tag{1}$$

where

$$A(\vartheta^\circ; q^{-1}) = 1 - \sum_{i=1}^{n} a_i^\circ q^{-i}$$

and

$$B(\vartheta^\circ; q^{-1}) = \sum_{i=0}^{m} b_{i+d}^\circ q^{-i}$$

are polynomials in the unit-delay operator q^{-1} and

$$\vartheta^\circ = [\, a_1^\circ \; a_2^\circ \dots a_n^\circ \; b_d^\circ \; b_{1+d}^\circ \dots b_{m+d}^\circ \,]^T$$

is the system parameter vector.

The process noise $\{n_t\}$ is subject to

Assumption 1 $\{n_t, \mathcal{F}_t\}$ is a martingale difference sequence with respect to an increasing sequence of σ-fields $\{\mathcal{F}_t\}$ and $\sup_t E[|n_{t+1}|^\alpha / \mathcal{F}_t] < \infty$, almost surely for some $\alpha > 2$. □

The system is assumed to satisfy the following

Assumption 2 Polynomials $q^s A(\vartheta^\circ; q^{-1})$ and $q^{s-d} B(\vartheta^\circ; q^{-1})$ are coprime, where $s = \max\{n, m+d\}$, □

which is known as 'controllability property' (see *e.g.* [3] and [6]).

Letting

$$\varphi_t = [\, y_t \dots y_{t-(n-1)} \; u_{t-(d-1)} \dots u_{t-(m+d-1)} \,]^T \tag{2}$$

be the observation vector, system (1) can be given the usual regression-like form

$$y_t = \varphi_{t-1}^T \vartheta^\circ + n_t, \tag{3}$$

to which we shall refer throughout the paper.

2.2 The standard least squares performance index

With reference to the plant representation (3), the least squares (LS) cost function is given by

$$V_t(\vartheta) = \sum_{s=0}^{t} (y_s - \varphi_{s-1}^T \vartheta)^2 \tag{4}$$

and its minimizer $\hat{\vartheta}_t^{LS}$ is the least squares estimate. It is well known that this estimate is consistent under some excitation condition. Consistency of LS has been studied in a huge number of papers. One of the most general results is supplied by Lai and Wei theorem ([1], Theorem 1). This result is stated below for future use.

Theorem 1 (*properties of* $\hat{\vartheta}_t^{LS}$)
*Suppose that the control law is causal (*i.e. u_t *is* $\mathcal{F}_t$*-measurable). Then,*

$$(\vartheta^\circ - \hat{\vartheta}_t^{LS})^T \sum_{s=0}^{t} \varphi_{s-1}\varphi_{s-1}^T (\vartheta^\circ - \hat{\vartheta}_t^{LS}) = O(\log \lambda_{max}(\sum_{s=0}^{t} \varphi_{s-1}\varphi_{s-1}^T)) \quad a.s., \tag{5}$$

which entails

$$\|\vartheta^\circ - \hat{\vartheta}_t^{LS}\|^2 = O\left(\frac{\log \lambda_{max}(\sum_{s=0}^{t} \varphi_{s-1}\varphi_{s-1}^T)}{\lambda_{min}(\sum_{s=0}^{t} \varphi_{s-1}\varphi_{s-1}^T)} \right) \quad a.s..$$

In particular, this implies that under the conditions

i) $\lambda_{min}(\sum_{s=0}^{t} \varphi_{s-1}\varphi_{s-1}^T) \to \infty$ *a.s.,*

ii) $\log \lambda_{max}(\sum_{s=0}^{t} \varphi_{s-1}\varphi_{s-1}^T) = o(\lambda_{min}(\sum_{s=0}^{t} \varphi_{s-1}\varphi_{s-1}^T))$ *a.s.,*

the least squares estimate is consistent. □

On the other hand, it is also well known ([2]-[9]) that the least squares estimated model is not necessarily controllable. Moreover, the norm of the estimate may not be bounded ([13]).

In the following section, we introduce a new performance index whose minimizer preserves the properties of $\hat{\vartheta}_t^{LS}$ stated in Theorem 1, but, in contrast with $\hat{\vartheta}_t^{LS}$, it results in a controllable model without requiring any particular excitation condition.

2.3 The penalized least squares performance index

For a given parameterization $\vartheta = [\, a_1 \; a_2 \ldots a_n \; b_d \; b_{1+d} \ldots b_{m+d}\,]^T$, a standard measure of controllability of the model $A(\vartheta;q^{-1})\, y_t = B(\vartheta;q^{-1})\, u_{t-d}$

is expressed by the absolute value of the *Sylvester resultant* associated with $q^s A(\vartheta; q^{-1})$ and $q^{s-d} B(\vartheta; q^{-1})$ given by the determinant of the Sylvester matrix ([14]):

$$Sylv(\vartheta) = \underbrace{\begin{bmatrix} 1 & & & & & & \\ -a_1 & 1 & & & & & \\ -a_2 & -a_1 & \ddots & & b_d & & \\ \vdots & -a_2 & \ddots & 1 & b_{1+d} & \ddots & \\ -a_s & \vdots & & -a_1 & \vdots & \ddots & b_d \\ & -a_s & & -a_2 & b_s & & b_{1+d} \\ & & \ddots & \vdots & & \ddots & \vdots \\ & & & -a_s & & & b_s \end{bmatrix}}_{s \quad\quad\quad\quad s},$$

where $a_i = 0$ for any $i > n$ and $b_i = 0$ for any $i > m + d$. As it is well known, see *e.g.* [15], $det(Sylv(\vartheta))$ is zero if and only if $q^s A(\vartheta; q^{-1})$ and $q^{s-d} B(\vartheta; q^{-1})$ have common factors. We exploit such a controllability measure in order to modify the least squares performance index so as to penalize uncontrollable or nearly uncontrollable models.

Specifically, we introduce a penalized performance index of the form

$$D_t(\vartheta) = V_t(\vartheta) + \alpha_t P(\vartheta), \tag{6}$$

where

$$P(\vartheta) = \frac{1}{|det(Sylv(\vartheta))|}$$

is the penalization term and $V_t(\vartheta)$ is the standard LS index given in equation (4). In the performance index (6) a major role is played by the scalar function α_t in front of the penalization term. In principle, this function should grow rapidly enough such that the penalization term $P(\vartheta)$ asserts itself. On the other hand, the penalization term $\alpha_t P(\vartheta)$ should be mild enough to avoid destroying the valuable properties of the least squares performance index stated in Theorem 1. The heart of the penalized least squares method lies on a suitable selection of α_t in such a way that the two contrasting objectives described above are met simultaneously. This makes the use of penalized techniques–which are well known in the field of operation research for the solution of constrained optimization problems (see *e.g.* [16])—attractive in the area of adaptive control. On the other hand, it should be noted that the penalized performance index (6) has, in general, multiple local minima. Therefore, $D_t(\vartheta)$ should be minimized by a global optimization algorithm (see *e.g.* [17], [18] and [19]). One can for instance resort to the multistart technique. In this method, a certain number of points ϑ are first selected (usually by means of a random procedure). Then, a standard local search method (such as a conjugate gradient method or a quasi-Newton method) initialized at the different points ϑ is run. The output of this procedure is a bunch of local minima. Among these, one finally select the minimum corresponding to the lower value for the function. Alternatively, one can resort to randomized algorithms such as simulated annealing, which seem to be more efficient than the multistart approach in the case of large dimension problems.

Denote by $\hat{\vartheta}_t$ the minimizer of the performance index $D_t(\vartheta)$:

$$\hat{\vartheta}_t := \arg \min_{\vartheta \in R^{n+m+1}} D_t(\vartheta) \tag{7}$$

(if $D_t(\vartheta)$ has more than one absolute minimum, any tie-breaking rule can be used).

Theorem 2 *(properties of $\hat{\vartheta}_t$)*
Assume that u_t is $\mathcal{F}_t$-measurable and select

$$\alpha_t := \log \lambda_{max}(\sum_{s=0}^{t} \varphi_{s-1}\varphi_{s-1}^T). \tag{8}$$

Then

i) the degree of coprimeness of polynomials

$$q^s A(\hat{\vartheta}_t; q^{-1})$$

and

$$q^{s-d} B(\hat{\vartheta}_t; q^{-1})$$

is a.s. bounded from below: $|det(Sylv(\hat{\vartheta}_t))| \geq c$, $\forall t$, where $c > 0$ is a suitable random constant;

ii) $(\vartheta^\circ - \hat{\vartheta}_t)^T \sum_{s=0}^{t} \varphi_{s-1}\varphi_{s-1}^T (\vartheta^\circ - \hat{\vartheta}_t) = O(\log \lambda_{max}(\sum_{s=0}^{t} \varphi_{s-1}\varphi_{s-1}^T))$ *a.s.*.

Proof: Observe first that equation

$$\sum_{s=0}^{t} \varphi_{s-1} y_s = [\sum_{s=0}^{t} \varphi_{s-1}\varphi_{s-1}^T] \hat{\vartheta}_t^{LS} \tag{9}$$

is easily derived from the definition $\hat{\vartheta}_t^{LS} = \arg \min_{\vartheta \in R^{n+m+1}} \sum_{s=0}^{t} (y_s - \varphi_{s-1}^T \vartheta)^2$.

Being $\hat{\vartheta}_t = \arg \min_{\vartheta \in R^{n+m+1}} D_t(\vartheta)$, we then have

$$
\begin{aligned}
&D_t(\hat{\vartheta}_t) - V_t(\hat{\vartheta}_t^{LS}) \leq D_t(\vartheta) - V_t(\hat{\vartheta}_t^{LS}) \\
&\quad = \textstyle\sum_{s=0}^{t}(y_s - \varphi_{s-1}^T \vartheta)^2 + \alpha_t P(\vartheta) - \sum_{s=0}^{t}(y_s - \varphi_{s-1}^T \hat{\vartheta}_t^{LS})^2 \\
&\quad = \vartheta^T \textstyle\sum_{s=0}^{t} \varphi_{s-1}\varphi_{s-1}^T \vartheta - 2\vartheta^T \sum_{s=0}^{t} \varphi_{s-1} y_s + \alpha_t P(\vartheta) \\
&\quad -(\hat{\vartheta}_t^{LS})^T \textstyle\sum_{s=0}^{t} \varphi_{s-1}\varphi_{s-1}^T \hat{\vartheta}_t^{LS} + 2(\hat{\vartheta}_t^{LS})^T \sum_{s=0}^{t} \varphi_{s-1} y_s,
\end{aligned}
$$

for all $\vartheta \in R^{n+m+1}$ and time instant t.

Substituting in this last expression equation (9), we obtain

$$
\begin{aligned}
&D_t(\hat{\vartheta}_t) - V_t(\hat{\vartheta}_t^{LS}) \leq \\
&\quad \vartheta^T \sum_{s=0}^{t} \varphi_{s-1}\varphi_{s-1}^T \vartheta - 2\vartheta^T \sum_{s=0}^{t} \varphi_{s-1}\varphi_{s-1}^T \hat{\vartheta}_t^{LS} + \alpha_t P(\vartheta) \\
&\quad -(\hat{\vartheta}_t^{LS})^T \sum_{s=0}^{t} \varphi_{s-1}\varphi_{s-1}^T \hat{\vartheta}_t^{LS} + 2(\hat{\vartheta}_t^{LS})^T \sum_{s=0}^{t} \varphi_{s-1}\varphi_{s-1}^T \hat{\vartheta}_t^{LS} \\
&= \quad (\vartheta - \hat{\vartheta}_t^{LS})^T \sum_{s=0}^{t} \varphi_{s-1}\varphi_{s-1}^T \, (\vartheta - \hat{\vartheta}_t^{LS}) + \alpha_t P(\vartheta), \; \forall \vartheta \in R^{n+m+1}, \; \forall t.
\end{aligned}
$$

By choosing $\vartheta = \vartheta^\circ$, this inequality can be rewritten as follows

$$
D_t(\hat{\vartheta}_t) - V_t(\hat{\vartheta}_t^{LS}) \leq (\vartheta^\circ - \hat{\vartheta}_t^{LS})^T \sum_{s=0}^{t} \varphi_{s-1}\varphi_{s-1}^T (\vartheta^\circ - \hat{\vartheta}_t^{LS}) + \alpha_t P(\vartheta^\circ), \; \forall \, t.
$$

Since $(\vartheta^\circ - \hat{\vartheta}_t^{LS})^T \sum_{s=0}^{t} \varphi_{s-1}\varphi_{s-1}^T (\vartheta^\circ - \hat{\vartheta}_t^{LS}) = O(\alpha_t)$ a.s. (see Theorem 1 and definition (8)) and $P(\vartheta^\circ)$ is finite (see Assumption 2), we conclude that

$$
\begin{aligned}
D_t(\hat{\vartheta}_t) - V_t(\hat{\vartheta}_t^{LS}) \quad &= \quad (\hat{\vartheta}_t - \hat{\vartheta}_t^{LS})^T \sum_{s=0}^{t} \varphi_{s-1}\varphi_{s-1}^T (\hat{\vartheta}_t - \hat{\vartheta}_t^{LS}) + \alpha_t P(\hat{\vartheta}_t) \\
&= \quad O(\alpha_t) \text{ a.s..} \qquad (10)
\end{aligned}
$$

From this, it is easily shown that $\limsup_{t\to\infty} P(\hat{\vartheta}_t) < \infty$, which means that there exists a (random) constant $k > 0$ such that $P(\hat{\vartheta}_t) = \dfrac{1}{\left|det(Sylv(\hat{\vartheta}_t))\right|} \leq k$, $\forall t$. By setting $c = \dfrac{1}{k}$, point *i)* is thereby proven.

As for point *ii)*, consider the inequality

$$
(\vartheta^\circ - \hat{\vartheta}_t)^T \sum_{s=0}^{t} \varphi_{s-1}\varphi_{s-1}^T (\vartheta^\circ - \hat{\vartheta}_t)
$$

$$\leq 2\left\{(\vartheta^\circ - \hat{\vartheta}_t^{LS})^T \sum_{s=0}^{t} \varphi_{s-1}\varphi_{s-1}^T (\vartheta^\circ - \hat{\vartheta}_t^{LS}) \right.$$
$$\left. + (\hat{\vartheta}_t^{LS} - \hat{\vartheta}_t)^T \sum_{s=0}^{t} \varphi_{s-1}\varphi_{s-1}^T (\hat{\vartheta}_t^{LS} - \hat{\vartheta}_t)\right\}.$$

Since both the terms in the right-hand-side are a.s. $O(\alpha_t)$ (see equations (5) and (10)), we get $(\vartheta^\circ - \hat{\vartheta}_t)^T \sum_{s=0}^{t} \varphi_{s-1}\varphi_{s-1}^T (\vartheta^\circ - \hat{\vartheta}_t) = O(\alpha_t)$ a.s., that is point *ii)*. □

Theorem 2 suggests that the function α_t must be adaptively selected in the light of the value taken by the observation vectors $\{\varphi_s\}_{s=0,1,\ldots,t-1}$ and therefore is time varying. The fact that α_t is adaptively selected is not surprising. In fact, the least squares part $V_t(\vartheta)$ of the performance index $D_t(\vartheta)$ depends on the observation vectors generated by the system. On the other hand, the penalization part $\alpha_t P(\vartheta)$ must be well-scaled with respect to $V_t(\vartheta)$ such that the minimizer of $D_t(\vartheta)$ still preserves some good properties of the minimizer of $V_t(\vartheta)$ and, at the same time, $\alpha_t P(\vartheta)$ is not negligible with respect to $V_t(\vartheta)$. From this, we see that α_t being dependent of the observation vectors is quite a natural result.

In adaptive control applications, in addition to controllability, it is sometimes useful to secure that the estimate cannot escape to infinity. As a matter of fact, this property is not fulfilled by the standard least squares algorithm (see *e.g.* [13]). In the penalized least squares algorithm, the boundedness of the estimate can be forced by adding an extra term which penalizes parameterizations with large norm. This leads to considering the performance index

$$\bar{D}_t(\vartheta) = V_t(\vartheta) + \alpha_t\{P(\vartheta) + \vartheta^T Q\vartheta\}, \quad (Q = Q^T > 0). \tag{11}$$

Theorem 3 *Under the same assumptions as in Theorem 2 the minimizer $\bar{\vartheta}_t$ of $\bar{D}_t(\vartheta)$ in equation (11) satisfies conditions* i) *and* ii) *in Theorem 2 and, in addition, keeps bounded:*

iii) $\bar{\vartheta}_t$ is a.s. bounded: $\|\bar{\vartheta}_t\| \leq k$, $\forall t$, where $k < \infty$ is a suitable random constant.

□

The proof of Theorem 3 is entirely similar to the one of Theorem 2 and therefore omitted.

3 Adaptive Stabilization

In this section, the effectiveness of the proposed penalized least squares method is shown by the derivation of a general adaptive stabilization result.

We start by observing that, when a plant is known and controllable, many different and well-established control methods can be used for its stabilization. Among them, we mention pole-placement ([15]), infinite horizon LQ control ([20]), and receding horizon control ([21]), which are all control techniques suitable for possibly nonminimum phase systems. The regulation law stemming from each one of these control methods takes the form

$$u_t = S(\vartheta; q^{-1})\, y_t + R(\vartheta; q^{-1})\, u_t,$$

where polynomials $S(\vartheta; q^{-1}) = \sum_{i=0}^{\beta} s_i(\vartheta) q^{-i}$ and $R(\vartheta; q^{-1}) = \sum_{i=1}^{\gamma} r_i(\vartheta)\, q^{-i}$ assume different expressions depending on the particular control method at hand. For all control strategies the coefficients $\{s_i\}_{i=0}^{\beta}$ and $\{r_i\}_{i=1}^{\gamma}$ are continuous functions of ϑ in the region where ϑ corresponds to a controllable model (see the references [15]-[21]).

In adaptive control the system parameter vector ϑ° is not known. Then, it is standard to resort to the so-called certainty-equivalence principle ([22]) which amounts to combine an identification algorithm and a control technique by simply tuning the control law to the estimated system. The main goal of the present section is to prove that the certainty-equivalence approach is successful in stabilizing the unknown system whenever a control method able to stabilize a possibly nonminimum phase controllable system is coupled with the penalized least squares identification method introduced in the previous section.

We start with a standard observation in adaptive control concerning the time variability of the estimated system. Since the parameter estimate is time-varying and the control law is tuned to such an estimate, an adaptive control system is always a time-varying system. On the other hand, it is well known that, in the case of time-varying systems, guaranteeing a stability property at each time point for the "frozen dynamics" does not imply that the overall time-varying system has a stable dynamics. This basic problem can be circumvented by updating the estimate at a slower rate than the updating of the system variables. Such a strategy, known as estimate with freezing effect, is for instance exploited in [23] and [24].
Following this idea, we define

$$\vartheta_t = \begin{cases} \bar{\vartheta}_t, & \text{if } t = t_i,\ i = 0, 1, 2, \dots \\ \vartheta_{t-1}, & \text{otherwise,} \end{cases} \tag{12}$$

where the time instants $\{t_i\}$ are obtained by the recursive equation $t_{i+1} = t_i + T_i$ initialized with $t_0 = 0$ (recall that $\bar{\vartheta}_t$ is the minimizer of $\bar{D}_t(\vartheta)$

in equation (11)). The time interval T_i is chosen so as to stabilize the time-varying estimated system. This is explained next.

Consider the time-varying estimated system

$$\left\{ \begin{array}{l} y_t = [1 - A(\vartheta_t; q^{-1})]\, y_t + B(\vartheta_t; q^{-1})\, u_{t-d} \\ u_t = S(\vartheta_t; q^{-1})\, y_t + R(\vartheta_t; q^{-1})\, u_t \end{array} \right. . \tag{13}$$

By letting $x_t := [y_t \ldots y_{t-l+1}\, u_t \ldots u_{t-q+1}]^T$ with $l = \max\{\beta, n\}$, $q = \max\{\gamma, d+m\}$, system (13) can be given the state space representation

$$x_t = F(\vartheta_t)\, x_{t-1},$$

where

$$\mathrm{F}(\vartheta) = \left[\begin{array}{cccccc} a_1 & \cdots & a_l & b_1 & \cdots & b_q \\ 1 & & & & & \\ & \ddots & & & & \\ & & 0 & & & \\ s_0(\vartheta)a_1 + s_1(\vartheta) & \cdots & s_0(\vartheta)a_l + s_l(\vartheta) & s_0(\vartheta)b_1 + r_1(\vartheta) & \cdots & s_0(\vartheta)b_q + r_q(\vartheta) \\ & & & 1 & & \\ & & & & \ddots & \\ & & & & & 0 \end{array} \right], \tag{14}$$

with $a_i = 0$ if $i > n$, $s_i(\vartheta) = 0$ if $i > \beta$, $b_i = 0$ if $i < d$ or $i > d+m$, $r_i(\vartheta) = 0$ if $i > \gamma$.

Choose now a constant $\mu < 1$ (*contraction* constant). The time interval T_i is then defined as

$$T_i := \inf\{\tau \in Z_+ : \|F(\vartheta_{t_i})^\tau\| \leq \mu\} \tag{15}$$

(note that such a T_i exists since $\vartheta_{t_i} = \bar{\vartheta}_{t_i}$ corresponds to a controllable system - Theorem 3). In this way, the time-varying system (13) is kept constant until its dynamics is contracted by a factor μ, whence guaranteeing its stability.

The fact that T_i is selected so as to stabilize the estimated system can be intuitively motivated as follows. In adaptive control, the true system is not known. Consequently, in an attempt to stabilize the true system, one stabilizes the estimated model. This will eventually result in the stabilization of the true system, provided that the estimated model accurately describes the behavior of the true system, at least in the long run.

In Theorem 4 we prove that the control law tuned to the estimated parameter with freezing (12) is in fact able to stabilize the unknown true system. The proof of Theorem 4 is based on the following technical Lemma.

Lemma 1 *The autonomous system $x_t = F(\vartheta_t)\, x_{t-1}$ is a.s. exponentially stable, uniformly in time: $\|x_t\| \leq M \bar{\nu}^{t-t^*} \|x_{t^*}\|$, $\forall\, t, t^*$, $t^* \leq t$, where $M > 0$ and $0 < \bar{\nu} < 1$ are suitable random constants.*

Remark 1 Note that the qualifying feature of the above statement is that the stability is *exponential, uniformly in time*, the stability of $x_t = F(\vartheta_t)x_{t-1}$ being already secured by the selection of T_i as stated in (15). $\Box$

Proof: We start by proving that, for any given real number $p > 0$, $T(\vartheta) := \inf\{\tau \in Z_+ : \|F(\vartheta)^\tau\| \leq \mu\}$ is uniformly bounded in the compact set $\mathcal{A}_p := \{\vartheta \in R^{n+m+1} : \|\vartheta\| \leq p^{-1}$ and $|det(Sylv(\vartheta))| \geq p\}$, *i.e.*

$$\sup_{\vartheta \in \mathcal{A}_p} T(\vartheta) < \infty. \tag{16}$$

Condition

$$|det(Sylv(\vartheta))| \geq p \tag{17}$$

implies that the system $y_t = \varphi_{t-1}^T \vartheta$ associated with parameter ϑ is controllable and therefore stabilized by the control law $u_t = S(\vartheta; q^{-1})\, y_t + R(\vartheta; q^{-1})\, u_t$. From this it follows that the dynamic matrix $F(\vartheta)$ of the time-invariant system

$$x_t = F(\vartheta)x_{t-1} \tag{18}$$

is exponentially stable.

Denote by $\{\lambda_i(\vartheta)\}_{i=1,\dots,l+q}$ the eigenvalues of $F(\vartheta)$. By the observation that $F(\vartheta)$ is a continuous function of ϑ, $\vartheta \in \mathcal{C} := \{\vartheta \in R^{n+m+1} :$ $q^s A(\vartheta; q^{-1})$ and $q^{s-d} B(\vartheta; q^{-1})$ are coprime$\}$, we have that $\rho(\vartheta) := \max_{i \in \{1,\dots,l+q\}} |\lambda_i(\vartheta)|$ is also a continuous function of ϑ, $\vartheta \in \mathcal{C}$. Being $\mathcal{A}_p$ compact and included in $\mathcal{C}$, the conclusion is drawn that $\rho := \max_{\vartheta \in \mathcal{A}_p} \rho(\vartheta) < 1$.

Fix now a real number $\nu \in (\rho, 1)$ and introduce the system

$$v_t = \frac{1}{\nu} F(\vartheta)\, v_{t-1}. \tag{19}$$

System (19) is exponentially stable $\forall \vartheta \in \mathcal{A}_p$, since $\left|\frac{\lambda_i(\vartheta)}{\nu}\right| \leq \frac{\rho}{\nu} < 1$, $\forall i$, $\forall \vartheta \in \mathcal{A}_p$. Hence, the solution $S(\vartheta)$ to the Lyapunov equation associated with matrix $\frac{1}{\nu} F(\vartheta)$

$$\frac{1}{\nu} F(\vartheta)^T S(\vartheta) \frac{1}{\nu} F(\vartheta) - S(\vartheta) = -I$$

is positive definite. Moreover, it is a standard fact that the state vector v_t of system (19) can be bounded in terms of $S(\vartheta)$ as follows

$$\|v_t\| \leq \sqrt{\frac{\lambda_{max}(S(\vartheta))}{\lambda_{min}(S(\vartheta))}}\, \| v_{\bar{t}}\|, \quad t \geq \bar{t}. \tag{20}$$

Since $S(\vartheta)$ is continuous in the closed set $\mathcal{A}_p$ (see [25]), we can define $c := \max_{\vartheta \in \mathcal{A}_p} \sqrt{\frac{\lambda_{max}(S(\vartheta))}{\lambda_{min}(S(\vartheta))}}$ and rewrite inequality (20) as $\|v_t\| \leq c\,\|v_{\bar{t}}\|, \;\; \forall\, t \geq \bar{t}, \forall\, \vartheta \in \mathcal{A}_p$. Setting $v_{\bar{t}} = x_{\bar{t}}$, we finally get a bound on the state vector x_t of the time-invariant system (18)

$$\|x_t\| \leq c\,\nu^{t-\bar{t}}\|x_{\bar{t}}\|, \quad t \geq \bar{t}, \forall\, \vartheta \in \mathcal{A}_p. \tag{21}$$

Set $\bar{T} = \inf\{\tau \in Z_+ : c\nu^\tau \leq \mu\}$. Since $\|x_{\bar{T}+\bar{t}}\| = \|F(\vartheta)^{\bar{T}}\, x_{\bar{t}}\| \leq \mu\|x_{\bar{t}}\|$, $\forall\, \vartheta \in \mathcal{A}_p$, $\forall\, x_{\bar{t}}$, then $\|F(\vartheta)^{\bar{T}}\| = \sup_{\|x\|\neq 0} \frac{\|F(\vartheta)^{\bar{T}}\, x\|}{\|x\|} \leq \mu$, $\forall\, \vartheta \in \mathcal{A}_p$ and therefore $T(\vartheta) = \inf\{\tau \in Z_+ : \|F(\vartheta)^\tau\| \leq \mu\}$ satisfies $T(\vartheta) \leq \bar{T}$, $\forall\, \vartheta \in \mathcal{A}_p$. This finally implies that

$$\sup_{\vartheta \in \mathcal{A}_p} T(\vartheta) \leq \bar{T} < \infty, \tag{22}$$

that is equation (16).

Let us turn to consider the time-varying system $x_t = F(\vartheta_t)\, x_{t-1}$.

From points *i)* and *iii)* of Theorem 3, it follows that there exists a (random) constant $\bar{p}$ such that $\bar{\vartheta}_t \in \mathcal{A}_{\bar{p}}$, $\forall t$. Denote by p the value of the random variable $\bar{p}$ on the outcome at hand. Being $\vartheta_t = \bar{\vartheta}_{t_i}, \;\; t \in [t_i, t_{i+1})$ (see (12)) and $\bar{\vartheta}_{t_i} \in \mathcal{A}_p, \;\; i \geq 0$, from equation (22) it follows that the updating time interval T_i in (15) is upper bounded:

$$T := \sup_{i \geq 0} T_i < \infty. \tag{23}$$

The thesis of the lemma is now derived from equations (23) and (21).

Denote by t_r the largest integer in the set $\{t_i\}$ lower than or equal to t and by t_s the smallest integer in $\{t_i\}$ greater than or equal to t^*. By applying equation (21) in the subintervals where ϑ_t keeps constant we obtain

$$\begin{aligned} \|x_t\| &\leq c\nu^{t-t_r+1}\|x_{t_r-1}\| \\ &\leq c\nu^{t-t_r+1}\mu^{r-s}\|x_{t_s-1}\| \\ &\leq c\nu^{t-t_r+1}\mu^{r-s}c\nu^{t_s-t^*-1}\|x_{t^*}\|. \end{aligned}$$

By letting $\bar{\nu} = \max\{\nu, \mu^{\frac{1}{T}}\}$, in view of equation (23), we finally obtain

$$\begin{aligned} \|x_t\| &\leq c^2\nu^{t-t_r+1}(\mu^{\frac{1}{T}})^{(r-s)T}\nu^{t_s-t^*+1}\|x_{t^*}\| \\ &\leq c^2\bar{\nu}^{t-t^*}\|x_{t^*}\| \end{aligned}$$

that is the thesis. □

On the basis of this Lemma, we can now prove the stability of the adaptively controlled system.

Theorem 4 (L^2-stability) *The closed-loop system*

$$\begin{cases} y_t = [1 - A(\vartheta^\circ; q^{-1})]\, y_t + B(\vartheta^\circ; q^{-1})\, u_{t-d} + n_t \\ u_t = S(\vartheta_t; q^{-1})\, y_t + R(\vartheta_t; q^{-1})\, u_t \end{cases}$$

is pathwise L^2-stable: $\displaystyle\limsup_{N\to\infty} \frac{1}{N} \sum_{t=1}^{N} \left[y_t^2 + u_t^2\right] < \infty$ *a.s.*.

Proof: Fix a time instant point $N > 0$.

In order to prove the thesis, we refer to Lemma 2 in the Appendix. According to the notations of this Lemma, we set $v_t := \vartheta^\circ - \vartheta_t$ and $z_t := \varphi_{t-1}$. With such positions, the hypotheses of Lemma 2 are satisfied, since $\{\vartheta^\circ - \vartheta_t\}$ is bounded and constant over $[t_i, t_i + T_i)$, $\forall i$ (see equation (12) and points *i)* and *iii)* in Theorem 3), there exists $T := \sup_i T_i$ (equation (23) in Lemma 1) and $\sum_{t=0}^{t_i} (\varphi_{t-1}^T(\vartheta^\circ - \vartheta_{t_i}))^2 = o(\sum_{t=0}^{t_i} \|\varphi_{t-1}\|^2) + O(1)$ (from point *ii)* in Theorem 3). Then, we have

$$\frac{1}{N} \sum_{t=0,\ t \notin \mathcal{B}_N}^{N} e_t^2 = \frac{1}{N} o(\sum_{t=0}^{N} \|\varphi_{t-1}\|^2 + N), \tag{24}$$

where $e_t := \varphi_{t-1}^T(\vartheta^\circ - \vartheta_t)$ and $\mathcal{B}_N$ is the set of instant points depending on N, whose cardinality is upper bounded by $(n + m + 1)T$ for any N (see Lemma 2 in the Appendix).

Now, observe that the time evolution of the state vector

$$x_t = [y_t \ldots y_{t-l+1}\, u_t \ldots u_{t-q+1}]^T$$

is governed by the equation

$$\begin{aligned} x_t &= F^\circ(\vartheta_t)\, x_{t-1} + G(\vartheta_t) n_t \\ &= F(\vartheta_t)\, x_{t-1} + G(\vartheta_t)[e_t + n_t], \end{aligned}$$

where

$$F^\circ(\vartheta) = \left[\begin{array}{cccccc} a_1^\circ & \ldots & a_l^\circ & b_1^\circ & \ldots & b_q^\circ \\ 1 & & & & & \\ & \ddots & & & & \\ & & 0 & & & \\ s_0(\vartheta)a_1^\circ + s_1(\vartheta) & \ldots & s_0(\vartheta)a_l^\circ + s_l(\vartheta) & s_0(\vartheta)b_1^\circ + r_1(\vartheta) & \ldots & s_0(\vartheta)b_q^\circ + r_q(\vartheta) \\ & & & 1 & & \\ & & & & \ddots & \\ & & & & & 0 \end{array}\right],$$

$G(\vartheta) = [1\,0 \ldots 0\, s_0(\vartheta)\, 0 \ldots 0]^T$ and $F(\vartheta)$ was defined in equation (14). In the sequel, we see vector x_t as generated according to

$$x_t = \begin{cases} F(\vartheta_t)\, x_{t-1} + G(\vartheta_t)[e_t + n_t], & t \notin \mathcal{B}_N \\ F^\circ(\vartheta_t)\, x_{t-1} + G(\vartheta_t) n_t, & t \in \mathcal{B}_N \end{cases}. \tag{25}$$

Since there exists a constant $p > 0$ such that ϑ_t belongs to the compact set $\mathcal{A}_p = \{\vartheta \in R^{n+m+1} : \|\vartheta\| \leq p^{-1}$ and $|det(Sylv(\vartheta))| \geq p\}$ (Theorem 3) and $F^\circ(\vartheta)$ and $G(\vartheta)$ are continuous function of ϑ, $\vartheta \in \mathcal{A}_p$ we then have that $\|F^\circ(\vartheta_t)\| \leq h$ and $\|G(\vartheta_t)\| \leq h$, h being a suitable constant. From this fact and the uniform exponential stability of the autonomous system $x_t = F(\vartheta_t)x_{t-1}$ (Lemma 1), it is easy to show that the state vector x_t generated by system (25) can be bounded as follows

$$\|x_t\| \leq (h\, M)^{|\mathcal{B}_N|} \left\{ \bar{\nu}^{t-|\mathcal{B}_N|} \|x_0\| + \right.$$
$$\left. \sum_{k=0}^{t} \bar{\nu}^{t-k-|\mathcal{B}_N|} |n_k| + \sum_{k=0, k \notin \mathcal{B}_N}^{t} \bar{\nu}^{t-k-|\mathcal{B}_N|} |e_k| \right\}, \;\; t \leq N.$$

As a consequence, we also have

$$\|x_t\|^2 \leq k_1 \left\{ \bar{\nu}^{2t} \|x_0\| + \sum_{k=0}^{t} \bar{\nu}^{t-k} n_k^2 + \sum_{k=0, k \notin \mathcal{B}_N}^{t} \bar{\nu}^{t-k} e_k^2 \right\} \;\; t \leq N,$$

k_1 being a suitable constant, which is independent of N.

Bearing in mind the definition of the observation vector φ_t, from this last inequality we get

$$\frac{1}{N} \sum_{t=0}^{N} \|\varphi_t\|^2 \leq k_2 \left\{ \frac{1}{N} \|x_0\| + \frac{1}{N} \sum_{t=0}^{N} n_t^2 + \frac{1}{N} \sum_{t=0, t \notin \mathcal{B}_N}^{N} e_t^2 \right\}$$

where k_2 is a suitable constant.

The first term in the right-hand-side of this expression vanishes as N tends to infinity. As for the second term $\frac{1}{N}\sum_{t=0}^{N} n_t^2$, by exploiting Chow's theorem (see *e.g.* [26] - Theorem 2.7 at page 36) and Assumption 1 it can be easily shown that it is almost surely bounded. Therefore, by using these estimates and applying inequality (24), we obtain

$$\frac{1}{N} \sum_{t=0}^{N} \|\varphi_t\|^2 = O(1) + o(\frac{1}{N} \sum_{t=0}^{N} \|\varphi_{t-1}\|^2),$$

which implies that $\frac{1}{N}\sum_{t=0}^{N} \|\varphi_t\|^2$ remains bounded. Then, the thesis immediately follows. $\square$

4 Conclusions

In this paper we have introduced a new method for the identification of discrete time linear systems affected by additive white noise. Such a method can be safely used in the adaptive control context, since it ensures the controllability of the identified model and, in addition, retains the closed-loop identification properties of the least squares estimate. As an example of application, we have proposed a certainty-equivalent adaptive regulation scheme based on the new parameter estimator, and we have shown that it ensures stability under general assumptions.

In the present formulation, our identification method is non-recursive. However, one can conceive a way to recursively minimize the penalized performance index. This requires further work.

Appendix

Lemma 2 *Consider a sequence of l-dimensional vectors $\{v_t\}$ such that the following assumptions are satisfied:*

i) *$\{v_t\}$ is bounded: $\|v_t\| \leq \bar{v}$, $\forall t$;*

ii) *$\{v_t\}$ is piecewise constant: $v_t = v_{t_i}$, $t \in [t_i, t_{i+1})$, where t_i is such that $T := \sup_i (t_{i+1} - t_i) < \infty$.*

Given a second l-dimensional vector sequence $\{z_t\}$ such that

iii) $\displaystyle\sum_{t=0}^{t_i} (z_t^T v_{t_i})^2 = o(\sum_{t=0}^{t_i} \|z_t\|^2) + O(1)$,

it follows that

$$\sum_{t=0,\ t \notin \mathcal{B}_N}^{N} (z_t^T v_t)^2 = o(\sum_{t=0}^{N} \|z_t\|^2 + N),$$

where $\mathcal{B}_N$ is a set of instant points which depends on N, whose cardinality, however, is upper bounded by Tl for any N: $|\mathcal{B}_N| \leq Tl$, $\forall N$.

Proof: Fix a real number $\epsilon > 0$ and a time instant N.

Consider the set of instant points in the interval $[0, N]$ where v_t changes: $t_0, t_1, \ldots, t_{i(N)}$, where $i(N) := \max\{i : t_i \leq N\}$. In these instant points we define a set of subspaces $\{S_{t_i}\}_{i=0}^{i(N)}$ through the following backward recursive procedure:
for $i = i(N) + 1$, set $S_i = \emptyset$

for $i = i(N), i(N)-1, \ldots, 0$, set (here and throughout the symbol $v_{t,S}$ stands for the projection of vector v_t onto the subspace S)

$$S_{t_i} = \begin{cases} S_{t_{i+1}}, & \text{if } \|v_{t_i, S^{\perp}_{t_{i+1}}}\| \leq \epsilon \\ S_{t_{i+1}} \oplus span\{v_{t_i}\}, & \text{otherwise.} \end{cases} \tag{26}$$

For each $t \in [0, N]$, with the notation $i(t) := \max\{i : t_i \leq t\}$, we have

$$(z_t^T v_t)^2 \leq 2\,(z^T_{t,S^{\perp}_{t_{i(t)}}} v_{t,S^{\perp}_{t_{i(t)}}})^2 + 2\,(z^T_{t,S_{t_{i(t)}}} v_{t,S_{t_{i(t)}}})^2. \tag{27}$$

By definition (26), the first term in the right-hand-side can be upper bounded as follows

$$(z^T_{t,S^{\perp}_{t_{i(t)}}} v_{t,S^{\perp}_{t_{i(t)}}})^2 \leq \epsilon^2 \|z_t\|^2 \tag{28}$$

To handle the second term, we first work out a basis in $S_{t_{i(t)}}$. For this purpose, consider the subset $\{\tau_j\}_{j=1}^{dim(S_{t_0})}$ of instant points $\{t_i\}_{i=0}^{i(N)}$ such that subspace S_{t_i} enlarges: $S_{\tau_j} \supset S_{t_i}$, $t_i > \tau_j$. The searched basis is $\{v_{\tau_j}\}_{j=dim(S_{t_0})-dim(S_{t_{i(t)}})+1}^{dim(S_{t_0})}$.

In view of the boundedness assumption *i)* and also considering the very definition of subspaces S_{t_i} (equation (26)), it is easy to see that vectors $\{v_{\tau_j}\}$ are spread in subspace $S_{t_{i(t)}}$ in such a way that the angle between each two of them tends to zero only when $\epsilon \to 0$. Consequently, there exists a constant $c(\epsilon)$, depending on ϵ, but independent of N, such that term $(z^T_{t,S_{t_{i(t)}}} v_{t,S_{t_{i(t)}}})^2$ in the right-hand-side of inequality (27) can be bounded as follows

$$\begin{aligned} (z^T_{t,S_{t_{i(t)}}} v_{t,S_{t_{i(t)}}})^2 &\leq \bar{v}^2 \|z_{t,S_{t_{i(t)}}}\|^2 \\ &\leq \bar{v}^2 c(\epsilon) \sum_{j=dim(S_{t_0})-dim(S_{t_{i(t)}})+1}^{dim(S_{t_0})} \|z_{t,span\{v_{\tau_j}\}}\|^2. \end{aligned} \tag{29}$$

By plugging estimates (28) and (29) in equation (27), we obtain

$$(z_t^T v_t)^2 \leq 2\epsilon^2 \|z_t\|^2 + 2\,\bar{v}^2\, c(\epsilon) \sum_{j=dim(S_{t_0})-dim(S_{t_{i(t)}})+1}^{dim(S_{t_0})} \|z_{t,span\{v_{\tau_j}\}}\|^2.$$

Summing up these relations from time $t = 0$ to $t = N$, we finally have

$$\sum_{t=0}^{N} (z_t^T v_t)^2 \leq 2\epsilon^2 \sum_{t=0}^{N} \|z_t\|^2 +$$

$$2\,\bar{v}^2\,c(\epsilon)\sum_{t=0}^{N}\sum_{j=dim(S_{t_0})-dim(S_{t_{i(t)}})+1}^{dim(S_{t_0})}\|z_{t,span\{v_{\tau_j}\}}\|^2. \tag{30}$$

Introduce now the time-varying set of instant points

$$\mathcal{B}_N := \cup_{j=1}^{dim(S_{t_0})}\{\tau_j, \tau_j+1, \ldots, \tau_j+T-1\}.$$

Since $dim(S_{t_0}) \leq l$, we obviously have $|\mathcal{B}_N| \leq Tl$.

Then,

$$\begin{aligned}
&\sum_{t=0,\ t\notin\mathcal{B}_N}^{N}\ \sum_{j=dim(S_{t_0})-dim(S_{t_{i(t)}})+1}^{dim(S_{t_0})}\|z_{t,span\{v_{\tau_j}\}}\|^2\\
&\quad\leq \sum_{j=1}^{dim(S_{t_0})}\sum_{t=0}^{\tau_j}\|z_{t,span\{v_{\tau_j}\}}\|^2\\
&\quad\leq \frac{1}{\epsilon^2}\sum_{j=1}^{dim(S_{t_0})}\sum_{t=0}^{\tau_j}(z_t^T v_{\tau_j})^2 \leq \frac{l}{\epsilon^2}[o(\sum_{t=0}^{N}\|z_t\|^2)+O(1)],
\end{aligned} \tag{31}$$

where the last inequality is a consequence of hypothesis *iii)* and the fact that $dim(S_{t_0}) \leq l$, $\forall N$.

By using inequality (30) and inequality (31), we obtain:

$$\begin{aligned}
\sum_{t=0,\ t\notin\mathcal{B}_N}^{N}(z_t^T v_t)^2 &\leq 2\epsilon^2\sum_{t=0}^{N}\|z_t\|^2 + 2\bar{v}^2 c(\epsilon)\frac{l}{\epsilon^2}[o(\sum_{t=0}^{N}\|z_t\|^2)+O(1)]\\
&\leq 2\epsilon^2 O(\sum_{t=0}^{N}\|z_t\|^2+N) + 2\bar{v}^2 c(\epsilon)\frac{l}{\epsilon^2}[o(\sum_{t=0}^{N}\|z_t\|^2+N)],
\end{aligned}$$

which finally implies that

$$\limsup_{N\to\infty}\frac{\displaystyle\sum_{t=0,\ t\notin\mathcal{B}_N}^{N}(z_t^T v_t)^2}{\displaystyle\sum_{t=0}^{N}\|z_t\|^2+N} \leq 2\epsilon^2.$$

Since ϵ is arbitrarily chosen, the thesis follows. □

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