CERTIFIED SYSTEM IDENTIFICATION

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TOWARDS DISTRIBUTION-FREE RESULTS

Marco C. Campi

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with:

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Erik Weyer

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thanks to :

- *Fabio Baronio*
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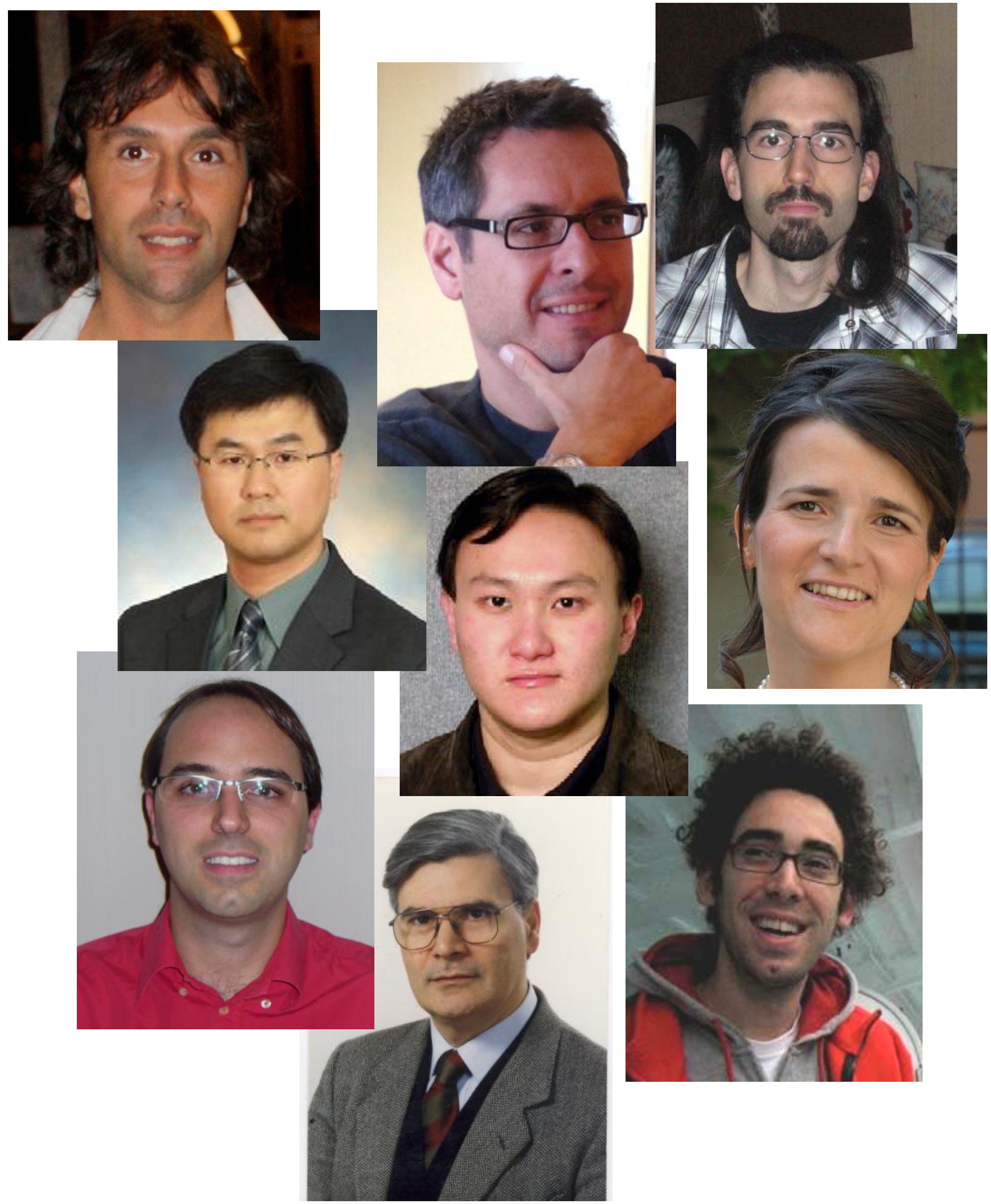

Preamble

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System identification is the science of constructing models from data

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A good system identification method should return models accompanied by certificates guaranteeing the quality of the model

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A good system identification method should return models accompanied by certificates guaranteeing the quality of the model

- $\|\theta^o - \widehat{\theta}\| \leq 0.1$
- $y_{t+1} \in$ region $\hat{Y}$ with probability 99%

$\vdots$

Preamble

System identification is the science of constructing models from data

and

A good system identification method should return models accompanied by certificates guaranteeing the quality of the model

Certificates are relevant to the practice of system identification, and are necessary for its scientific use

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System identification relies on data, and data is the real wealth in a system identification procedure

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therefore

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A system identification method should squeeze out all the relevant information contained in the data, for the purpose of constructing models and of certifying their quality

Preamble

This talk is about data, and the quality of models obtained using data

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N data points

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MODEL

N data points

CERTIFICATE of QUALITY

… a common thread

… a common thread

deterministic, probabilistic $\longrightarrow$ **distribution-free**

... a common thread

deterministic, probabilistic → **distribution-free**

probability exists,
but probability
values do not count

... a common thread

deterministic, probabilistic $\longrightarrow$ **distribution-free**

$$\boxed{\textit{we should look for distribution-free results}}$$

... a common thread

deterministic, probabilistic $\longrightarrow$ **distribution-free**

paradigm shift

A journey through:

- parameter estimation
- prediction
- filtering

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- parameter estimation
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an example

an example

$$y_t = \theta^o + n_t$$

an example

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10 independent observations

an example

$$y_t = \theta^o + n_t$$

t	1	2	3	4	5	6	7	8	9	10
y_t	0.56	−0.66	1.12	1.32	−0.14	2.25	−0.21	0.96	1.28	1.17

an example - deterministic

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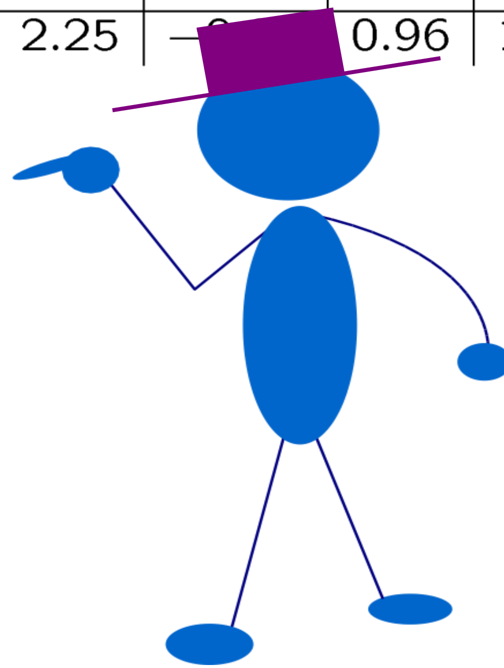

<u>Assumption:</u> $|n_t| \leq 2$

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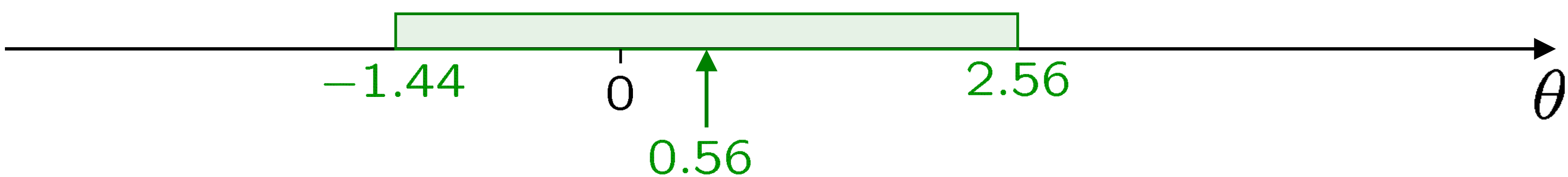

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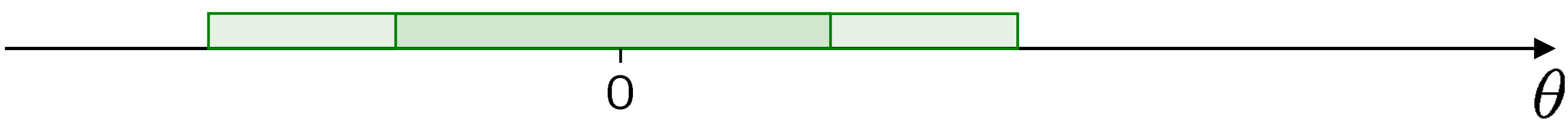

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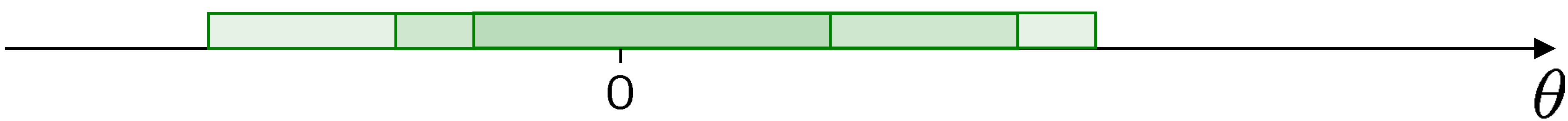

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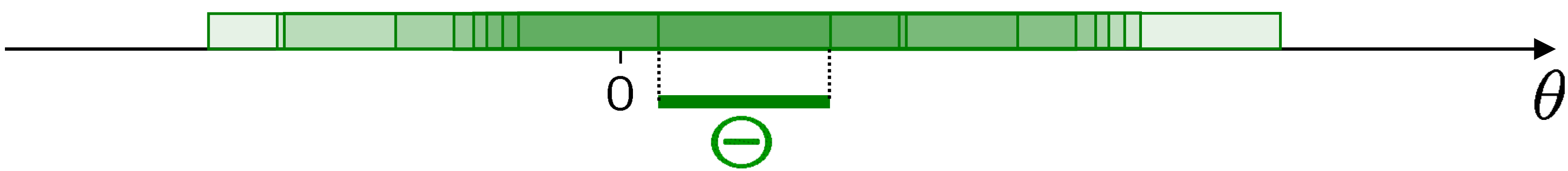

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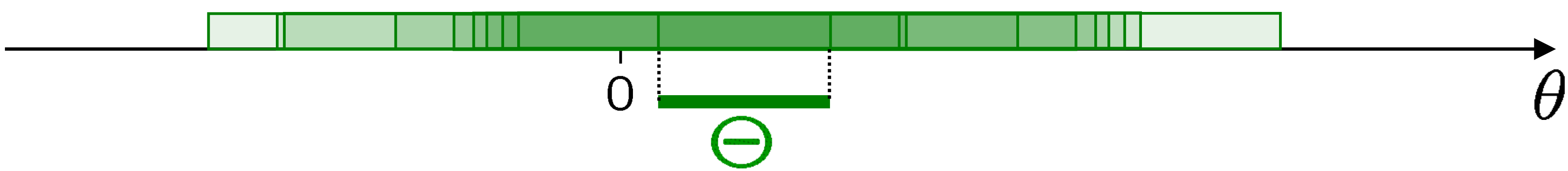

Claim: $\theta^o \in \Theta$ (deterministic)

an example - probabilistic

an example - probabilistic

are we sure that $|n_t| \leq 2$?

an example - probabilistic

$$y_t = \theta^o + n_t$$

<u>Assumption:</u> $n_t \sim \mathsf{Gaussian}(0, 1)$

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LS estimate: $\widehat{\theta}_{LS} = \frac{1}{10} \sum_{t=1}^{10} y_t = 0.76$

an example - probabilistic

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Assumption: $n_t \sim \text{Gaussian}(0, 1)$

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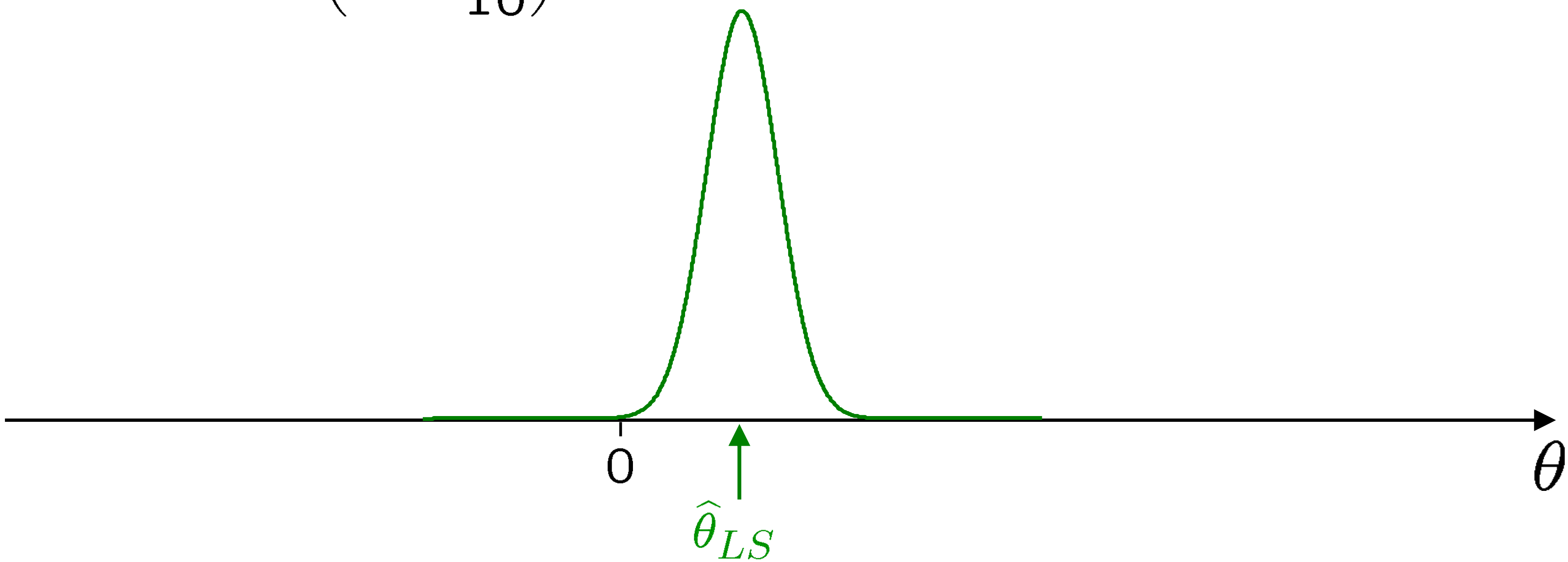
$$\theta^o - \widehat{\theta}_{LS} \sim \text{Gaussian}\left(0, \frac{1}{10}\right)$$

an example - probabilistic

Gaussian $\left(\widehat{\theta}_{LS}, \frac{1}{10}\right)$



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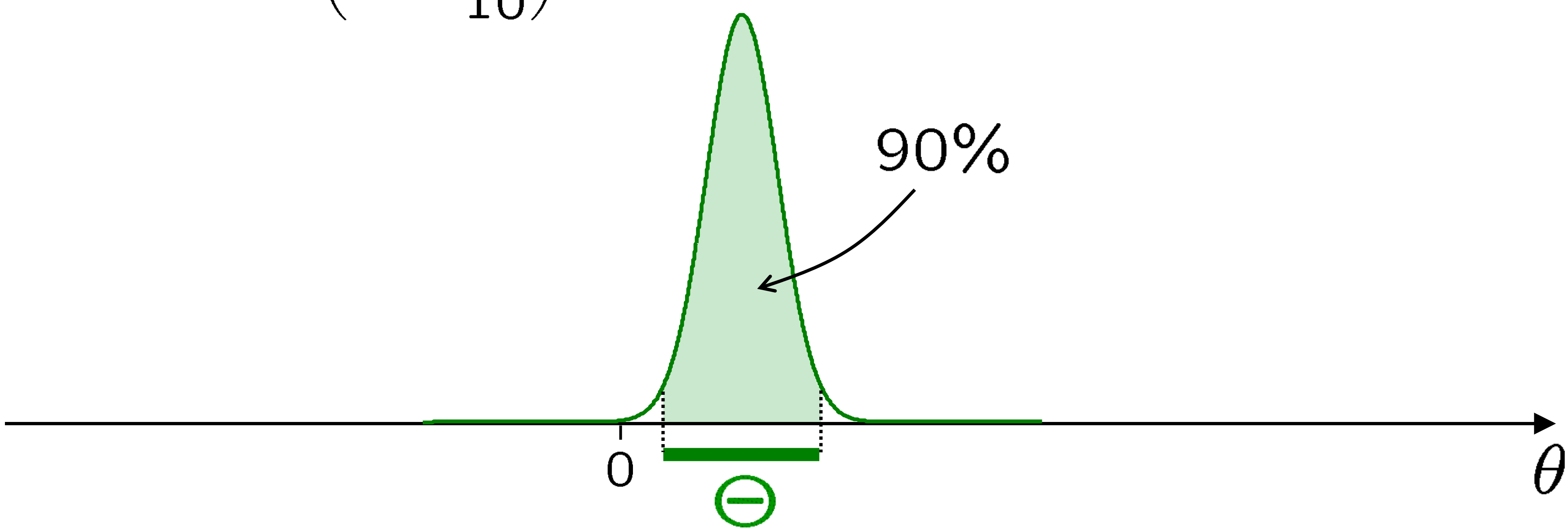

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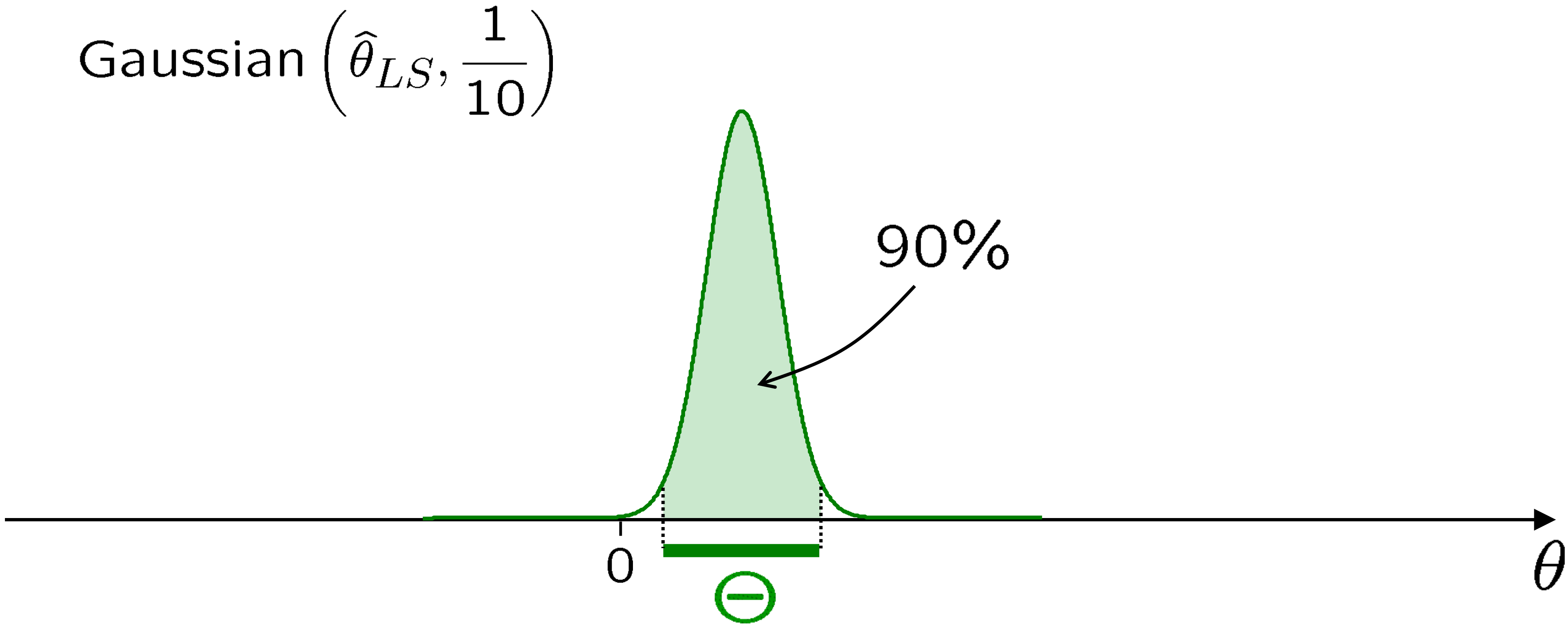


Claim: $\theta^o \in \Theta$ with probability 90%

are we sure that

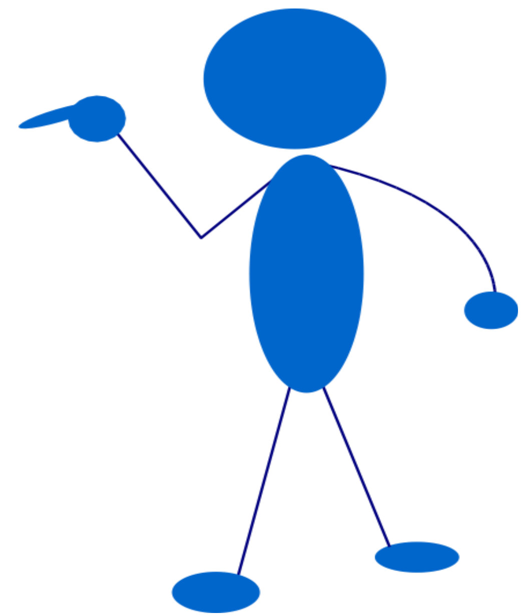
$n_t \sim \text{Gaussian}(0, 1)$?

a paradigm shift

an example - distribution-free (almost)

$$y_t = \theta^o + n_t$$

<u>Assumption:</u> n_t has an unknown distribution symmetric around zero



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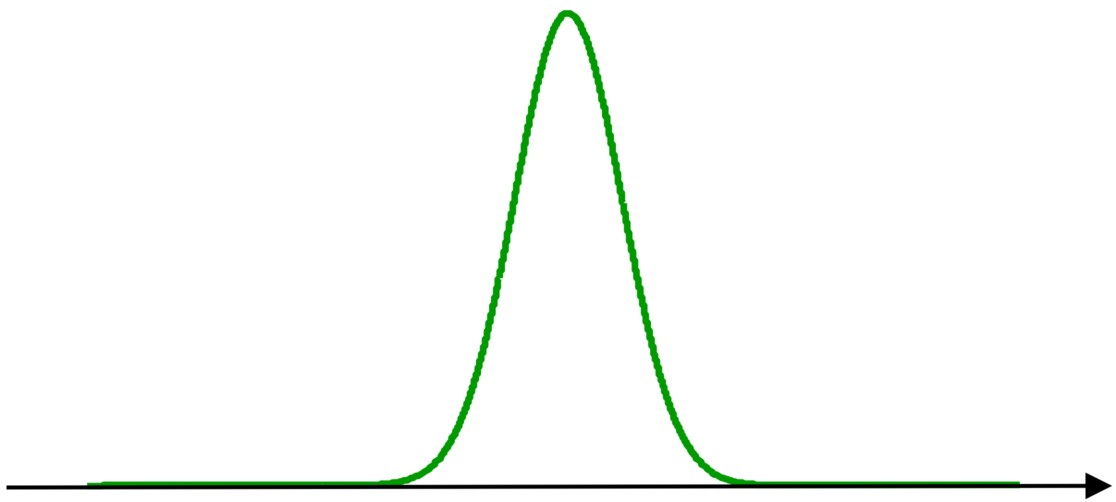

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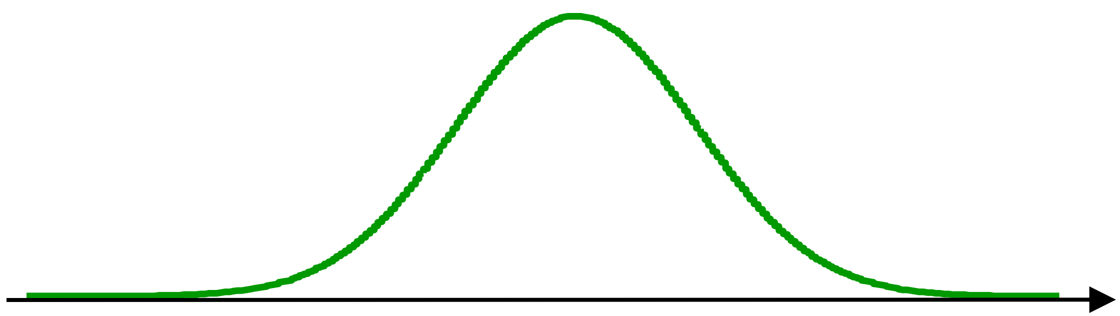

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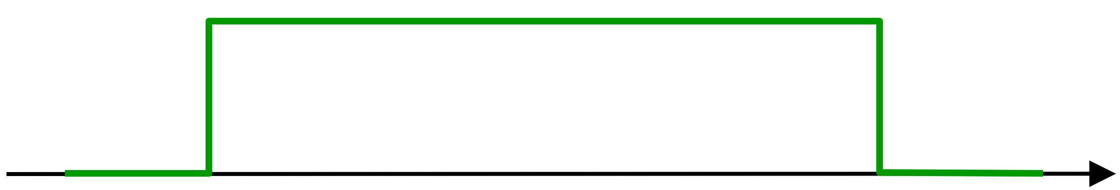

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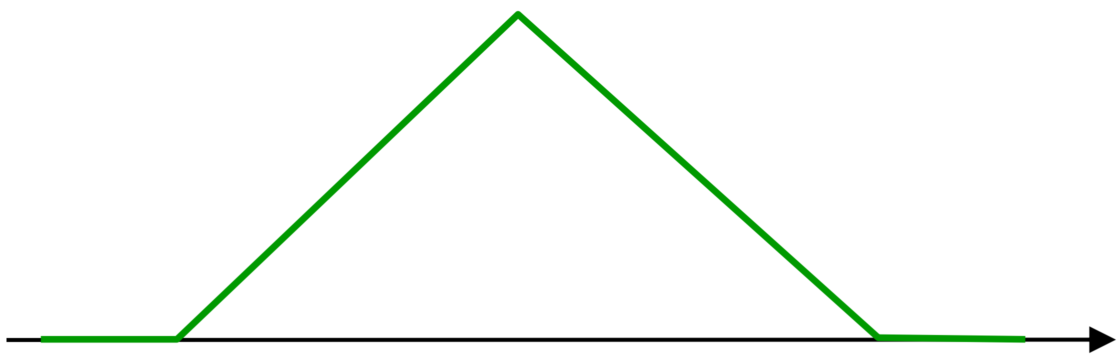

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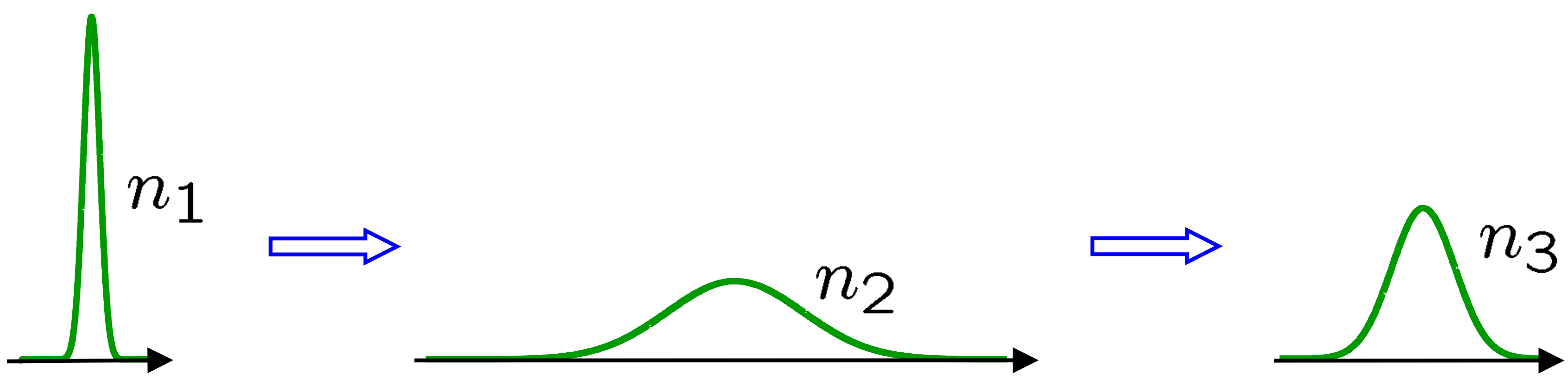

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$$\left[\sum_{t=1}^{10} (y_t - \theta)\right]^2 = \text{TEST PARABOLA}$$

an example - distribution-free (almost)

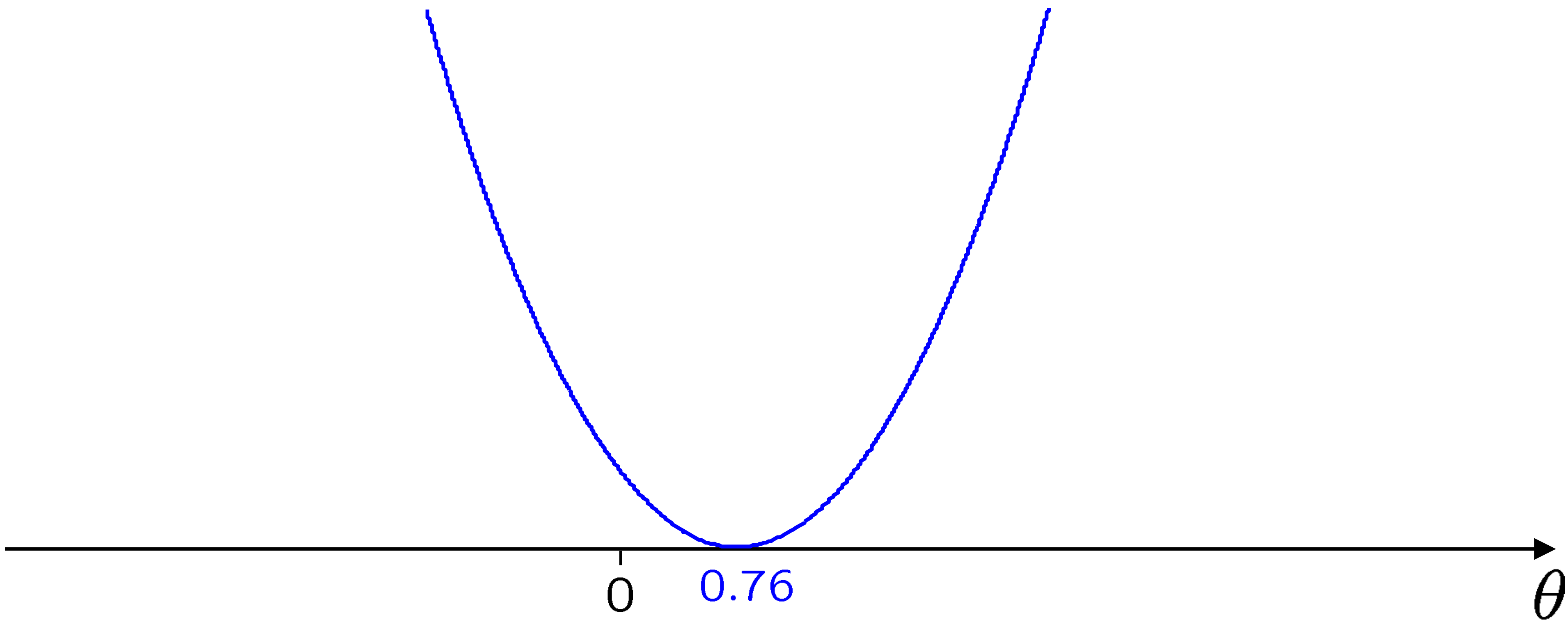
$$y_t = \theta^o + n_t$$

Assumption: n_t has an unknown distribution symmetric around zero

$$\left[\sum_{t=1}^{10} (y_t - \theta)\right]^2 = \text{TEST PARABOLA}$$

$$\text{vertex} = \frac{1}{10}\sum_{t=1}^{10} y_t = \hat{\theta}_{LS}$$

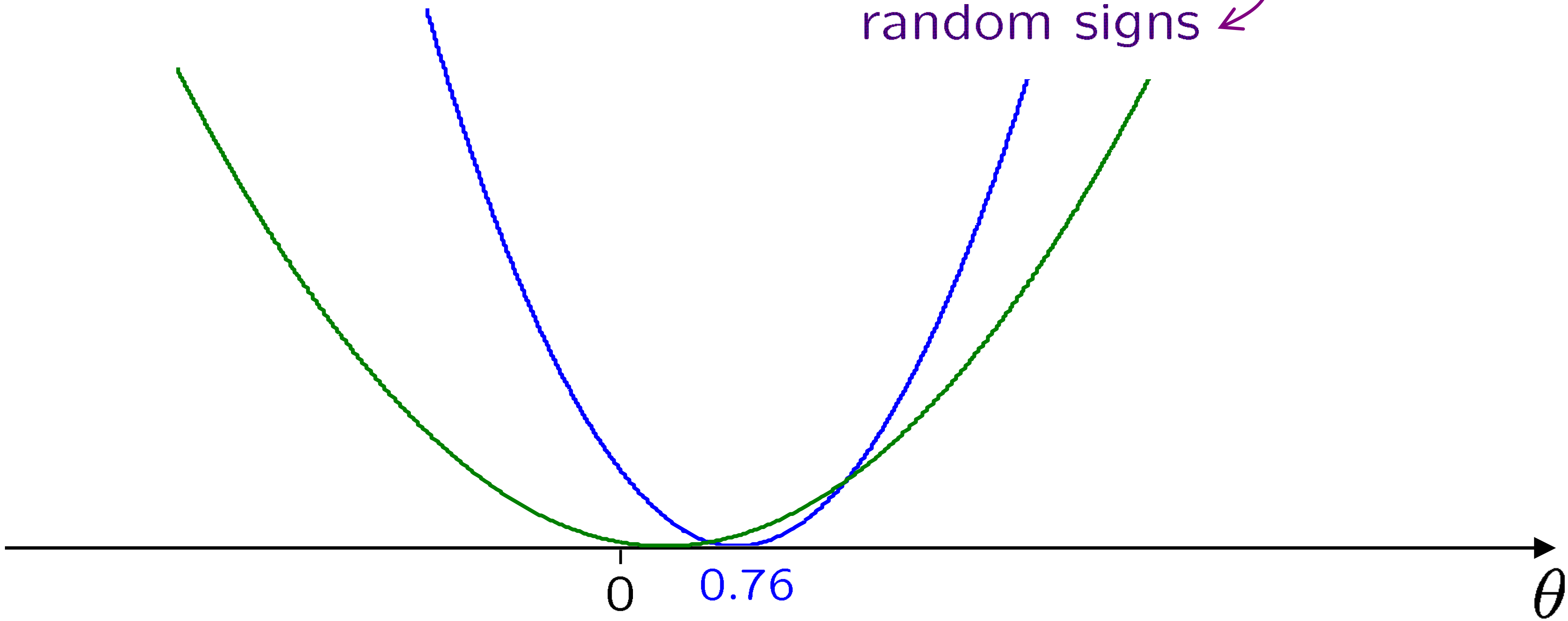
an example - distribution-free (almost)



an example - distribution-free (almost)

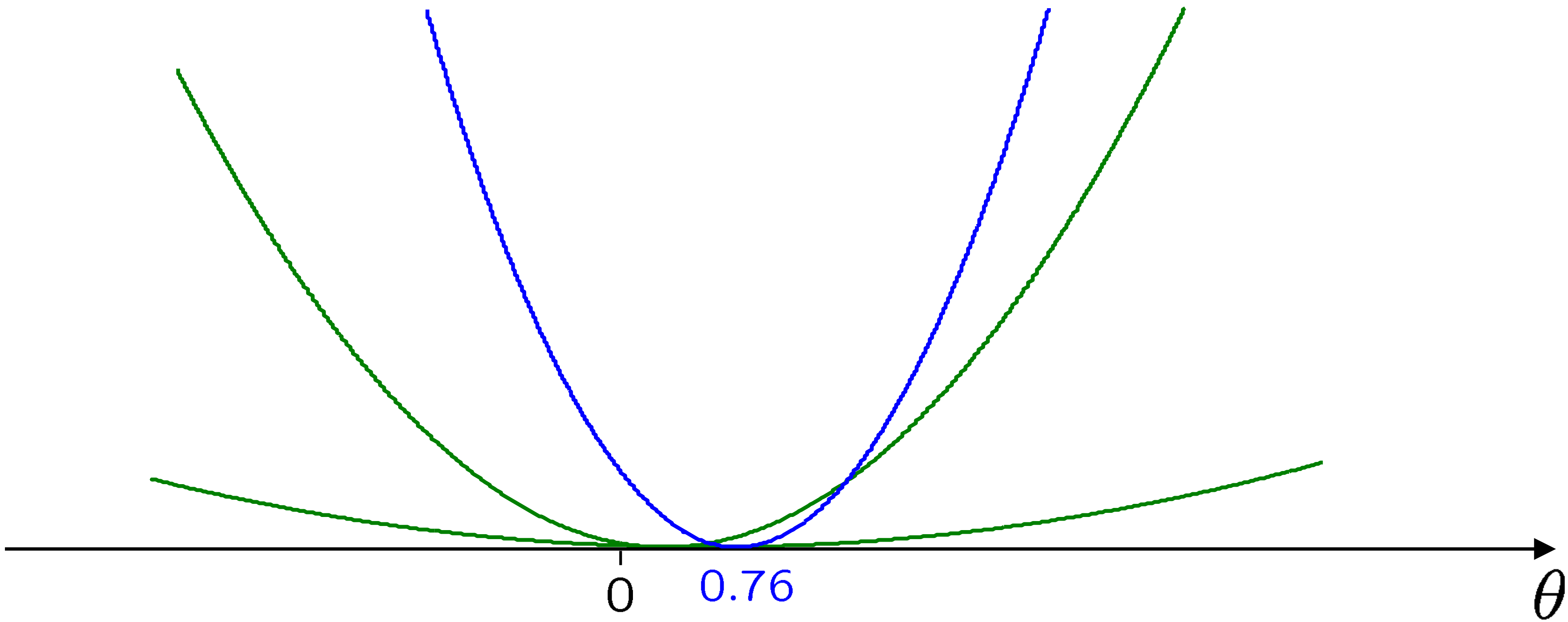
$$\text{PARABOLA} = \left[\sum_{t=1}^{10} \pm (y_t - \theta)\right]^2$$

random signs



an example - distribution-free (almost)

$$2^0 \text{ PARABOLA} = \left[\sum_{t=1}^{10} \pm(y_t - \theta)\right]^2$$



an example - distribution-free (almost)

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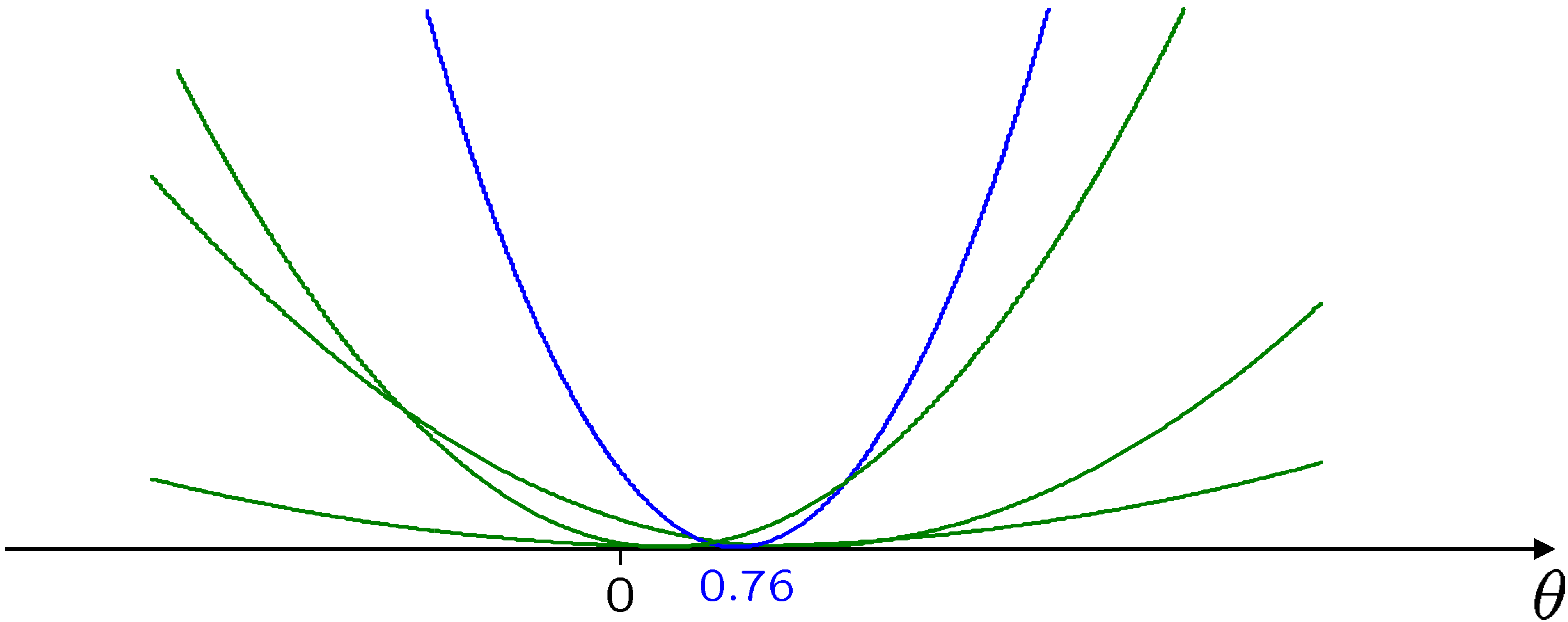
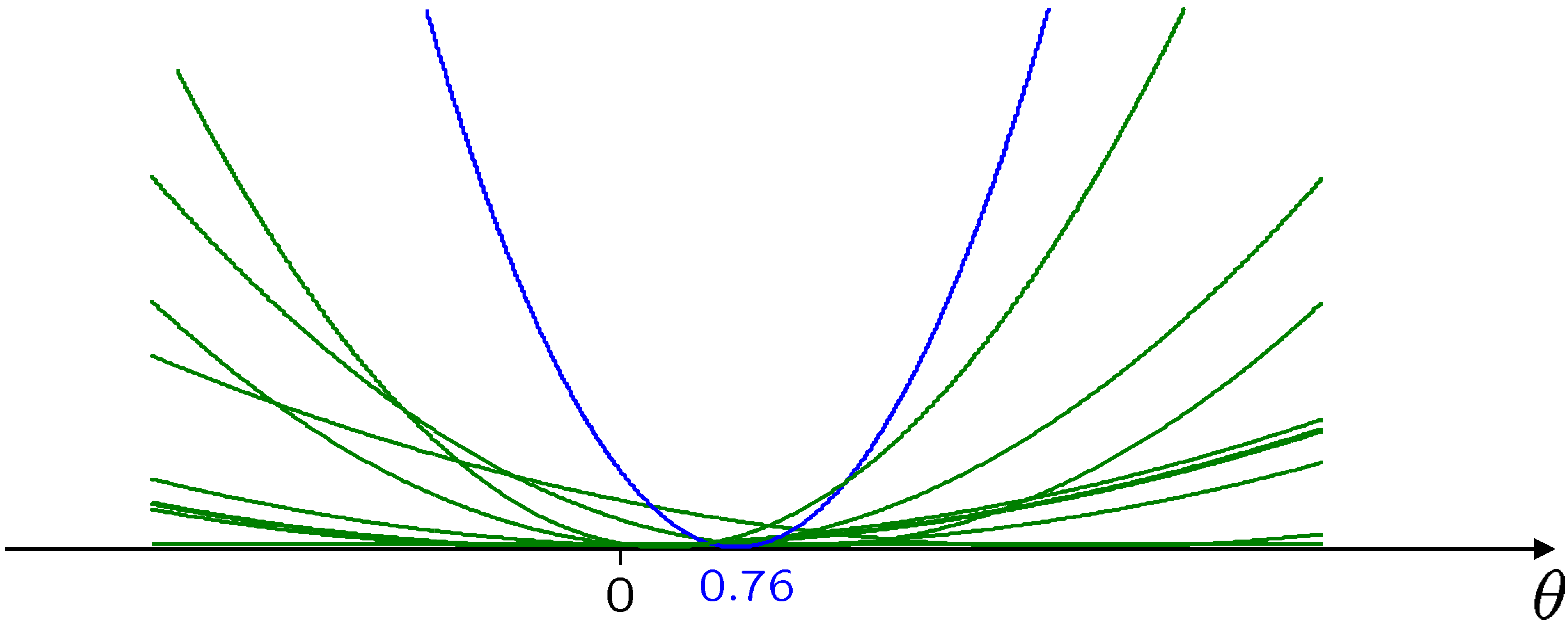


an example - distribution-free (almost)

$$9^0 \text{ PARABOLA} = \left[\sum_{t=1}^{10} \pm(y_t - \theta)\right]^2$$



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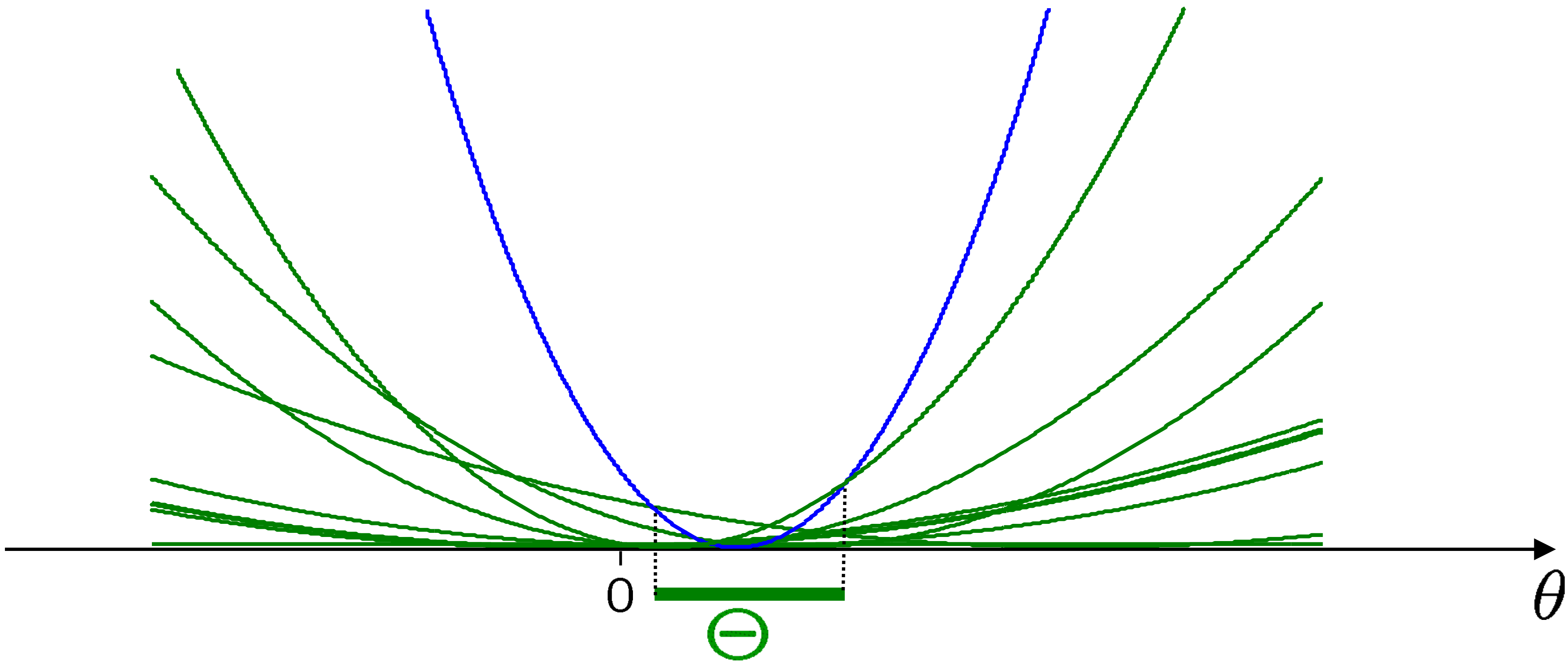

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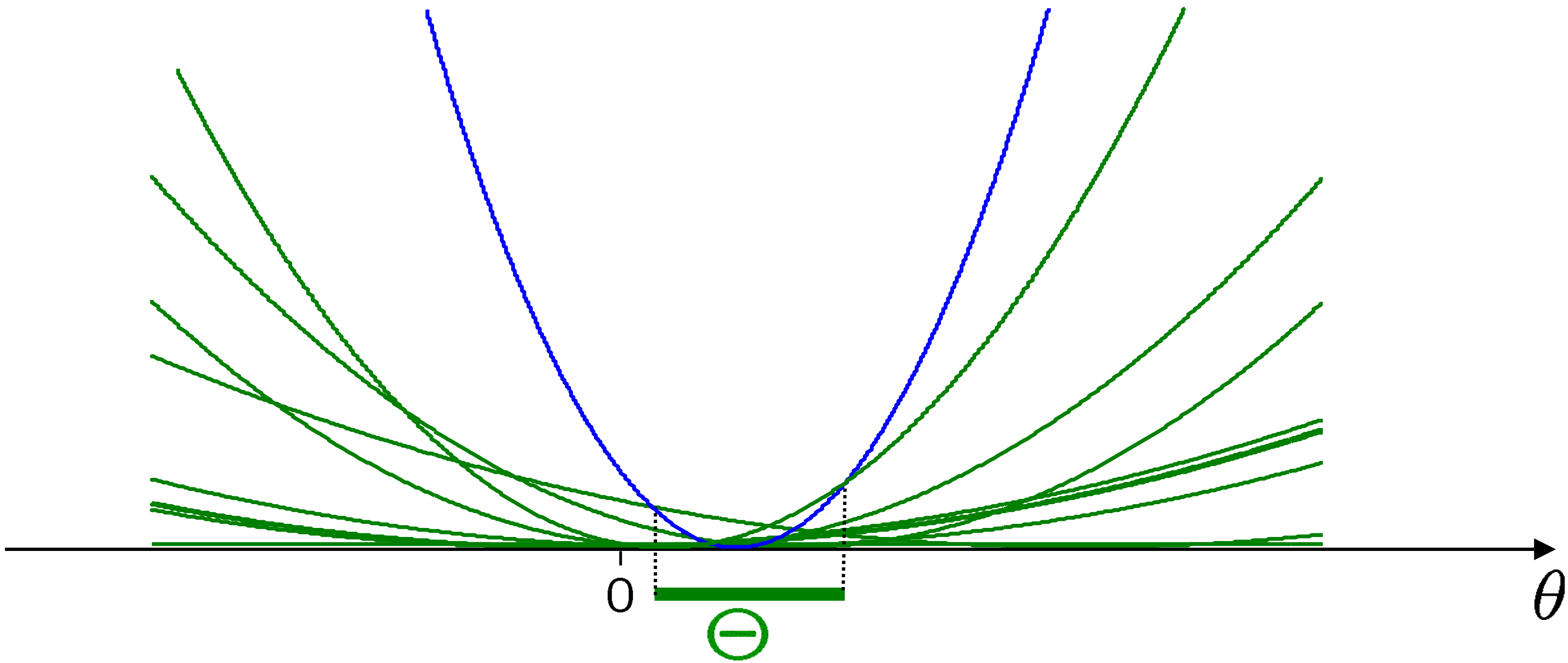

Claim: $\theta^o \in \Theta$ with probability 90%

Proof:

TEST PARABOLA($\theta = \theta^O$) $= \left[\sum^{10} \ldots)\right]^2 = \left[\sum_{t=1}^{10} (\theta^O + n_t - \theta^O)\right]^2$

$\ldots n_t \Big]^2$

1^0 PARABOLA($\theta = \ldots \left[\sum_{t=1} \pm n_t\right]^2$

all parabolas ... same probability to be at top

very simple!

there is a lot of information in the data that traditional methods do not exploit *there is a lot of information in the data that traditional methods do not exploit*

let the data speak!

SPS = Sign Perturbed Sums (with Csaji, Weyer)

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$$y_t = \theta_1^o u_{t-1} + \theta_2^o u_{t-2} + n_t$$

$$\text{TEST PARABOLA} = \left\| \sum_{t=1}^{N} \begin{bmatrix} u_{t-1} \\ u_{t-2} \end{bmatrix} (y_t - \theta_1 u_{t-1} - \theta_2 u_{t-2}) \right\|^2$$

$$\text{vertex} = \widehat{\theta}_{LS}$$

$$\text{n-th PARABOLA} = \left\| \sum_{t=1}^{N} \pm \begin{bmatrix} u_{t-1} \\ u_{t-2} \end{bmatrix} (y_t - \theta_1 u_{t-1} - \theta_2 u_{t-2}) \right\|^2$$

SPS = Sign Perturbed Sums (with Csaji, Weyer)

- $A^o(z^{-1})y_t = B^o(z^{-1})u_t + n_t$

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- $A^o(z^{-1})y_t = B^o(z^{-1})u_t + n_t$

- $y_t = G^o(z^{-1})u_t + H^o(z^{-1})n_t$

Example

$$y_t = 0.6u_{t-1} + 0.8u_{t-2} + n_t$$

$$u_t = 0.8u_{t-1} + v_t$$

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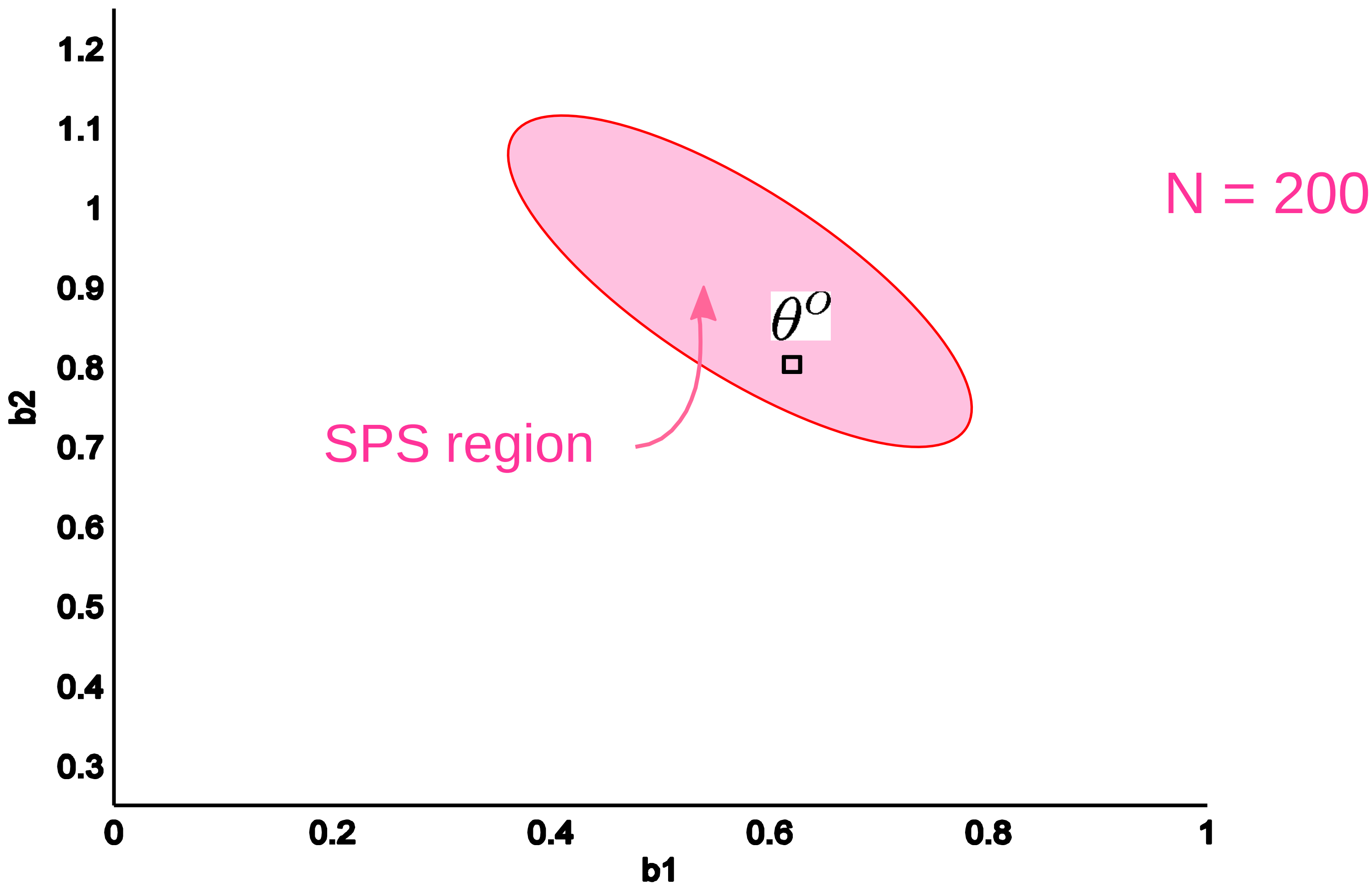

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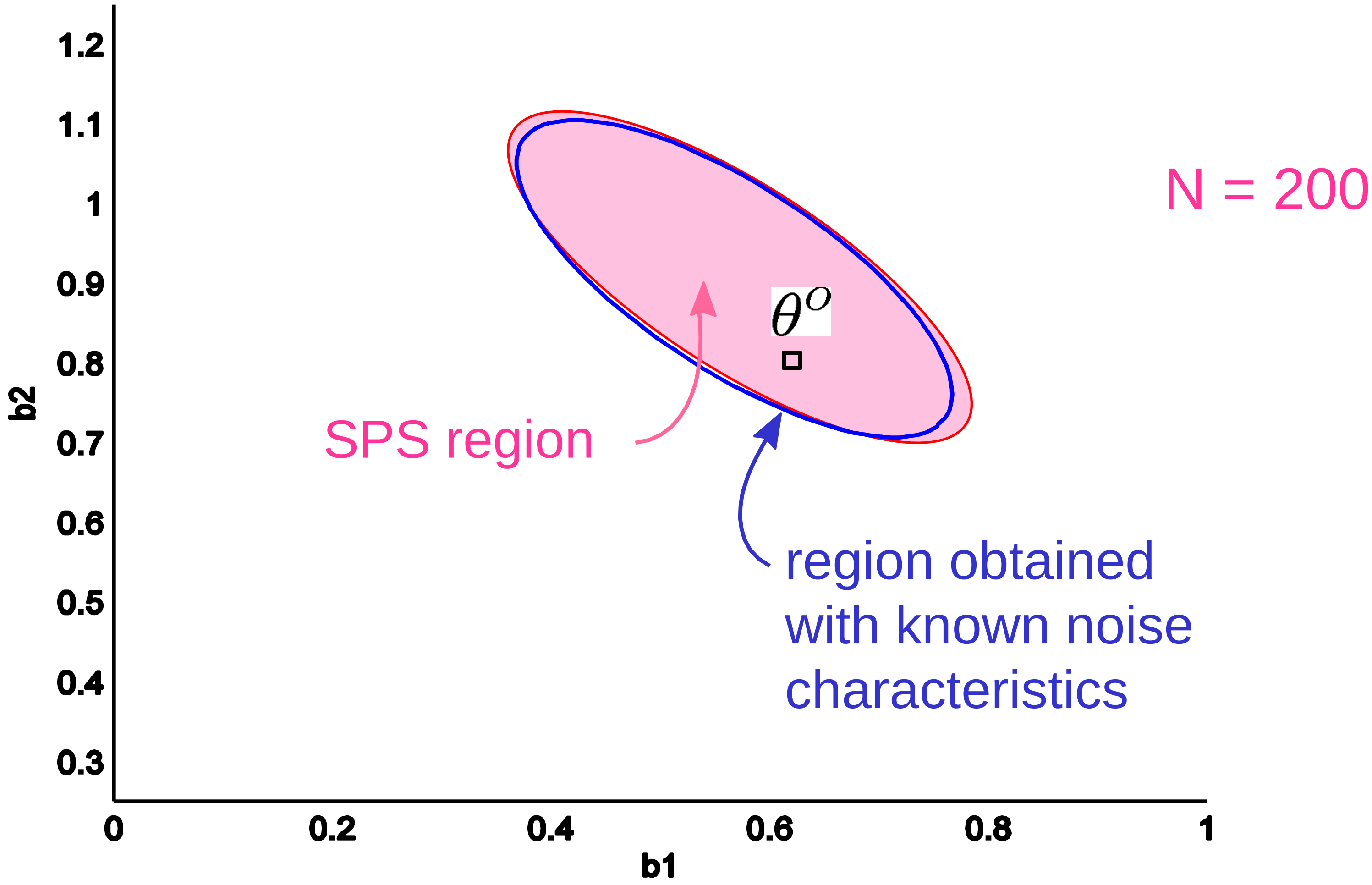

. . . mid-talk summary

deterministic

Claim: $\theta^o \in \Theta$

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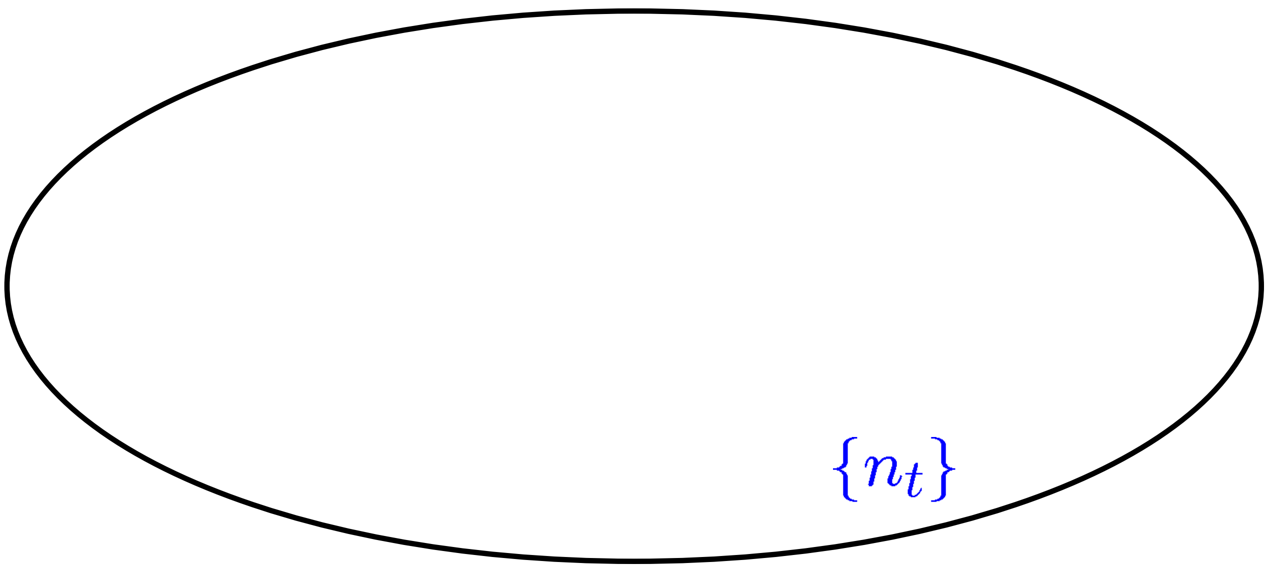
set-theoretic statement



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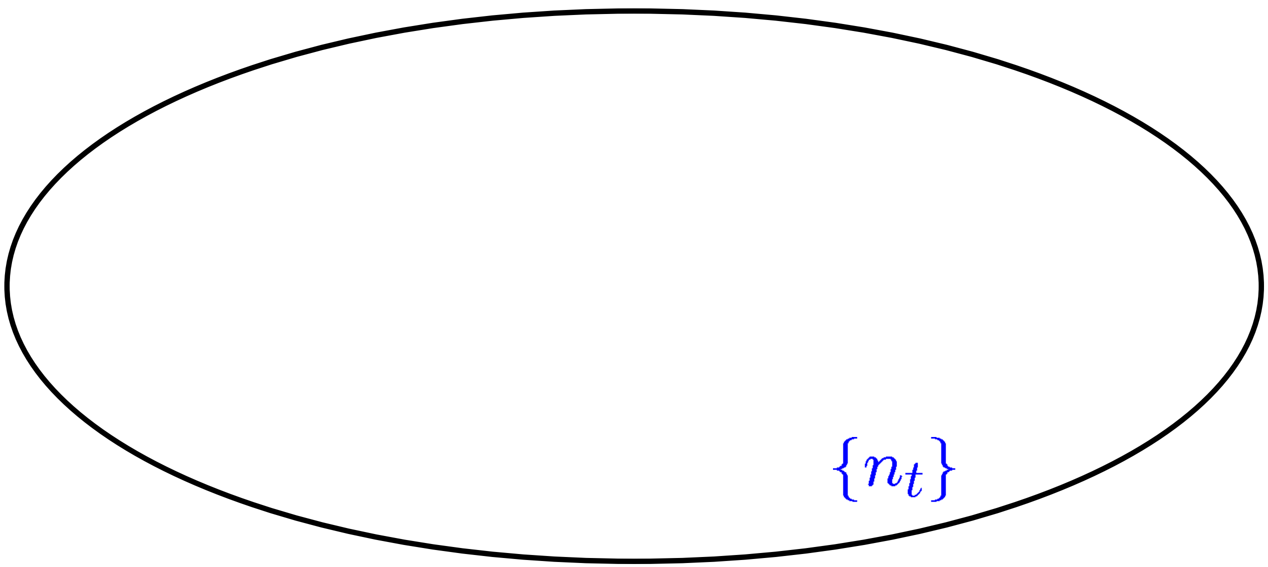
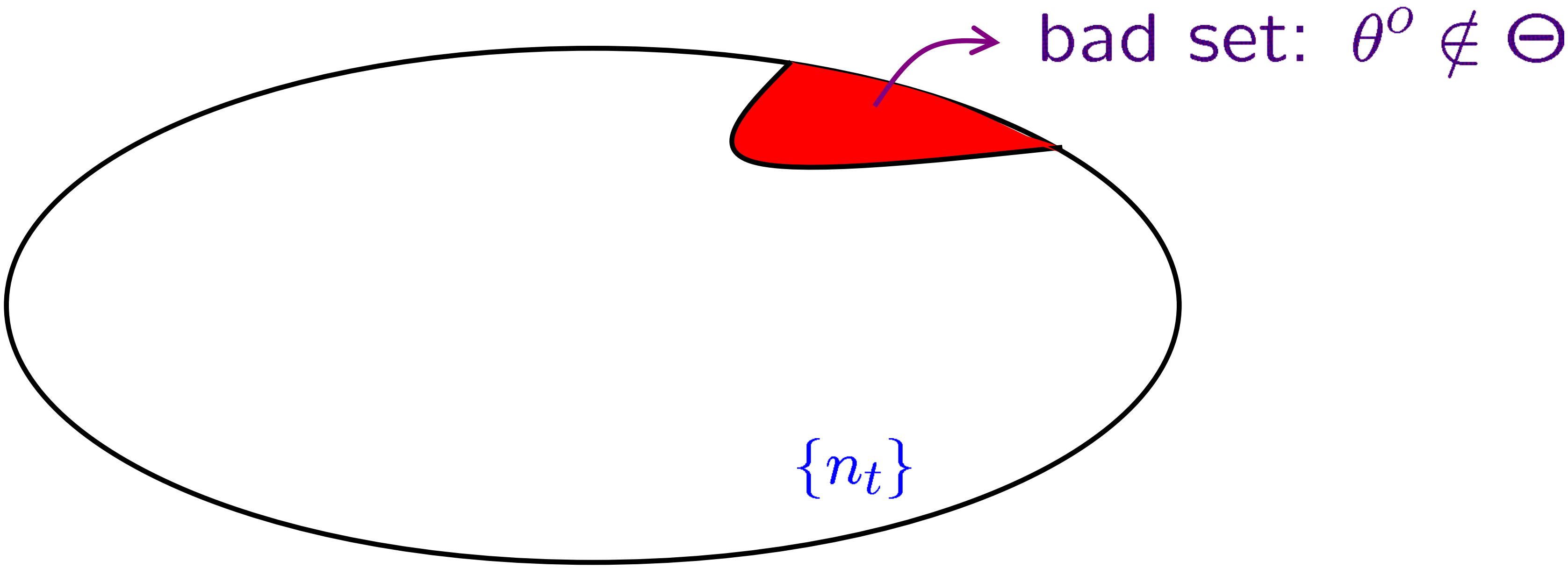


rigid deterministic assumptions

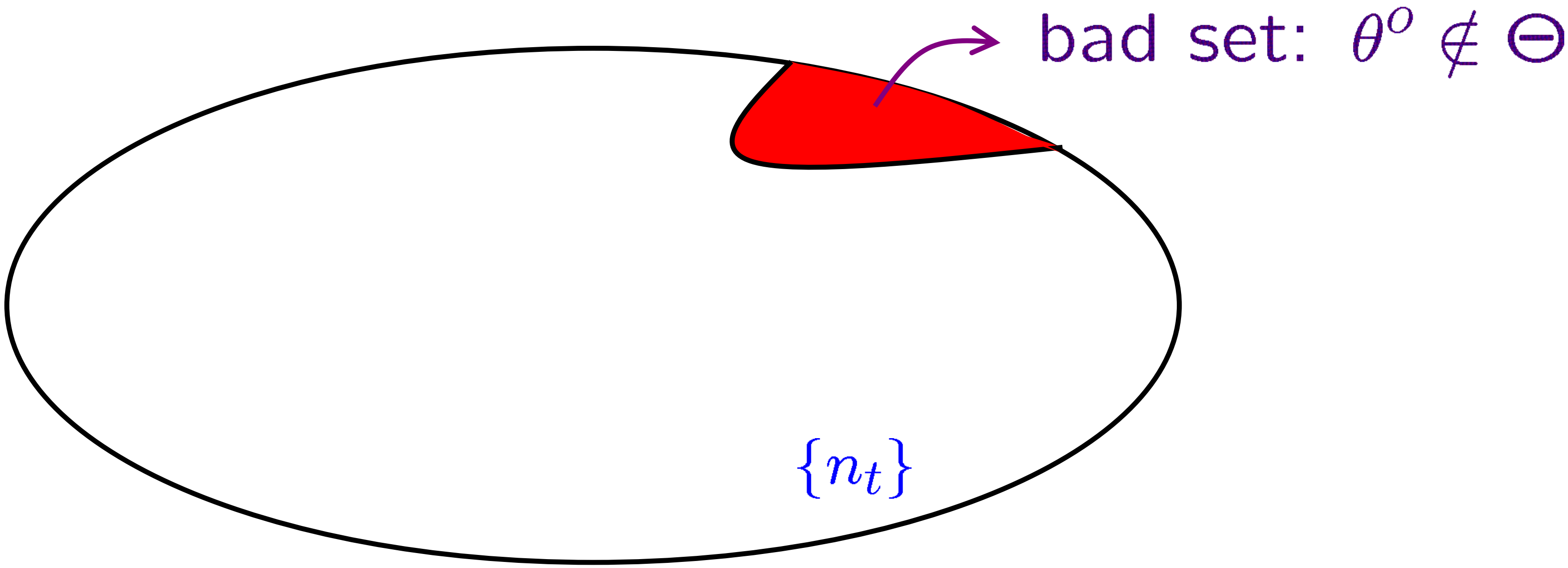
Mark Twain: “what gets us into trouble is not what we don't know. It's what we know for sure that just ain't so.”

remove the stringent deterministic assumptions

remove the stringent deterministic assumptions

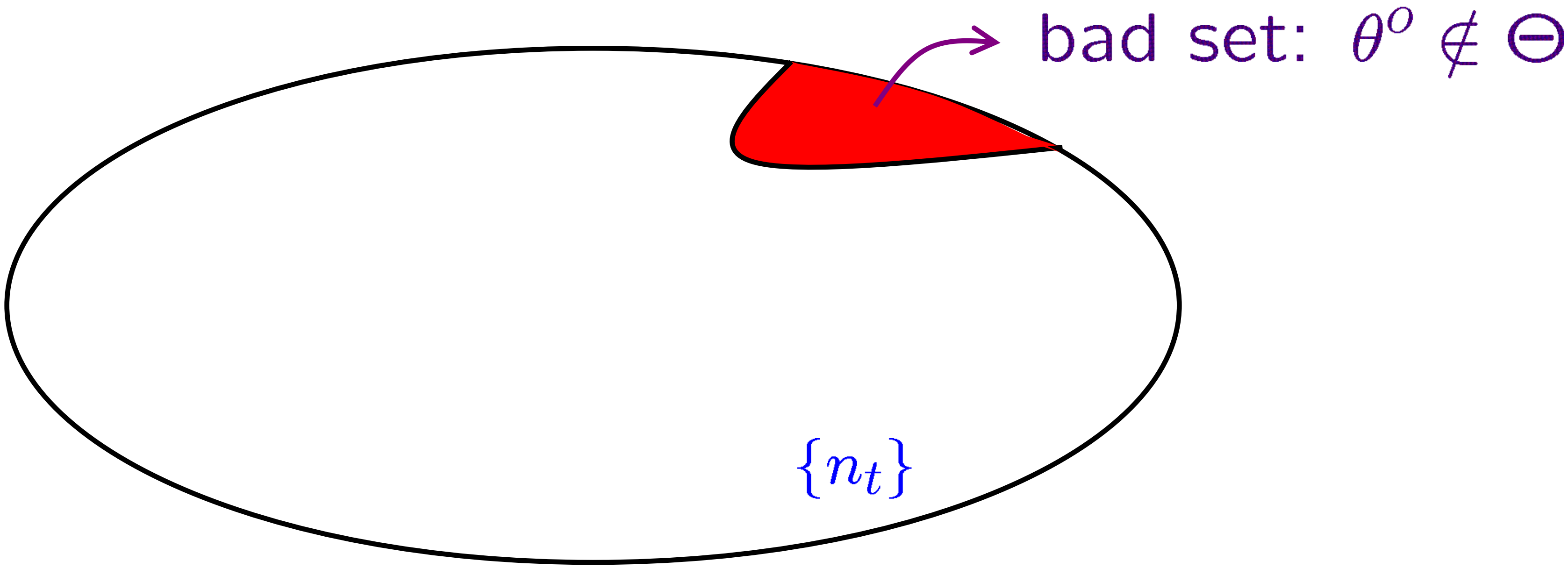


remove the stringent deterministic assumptions



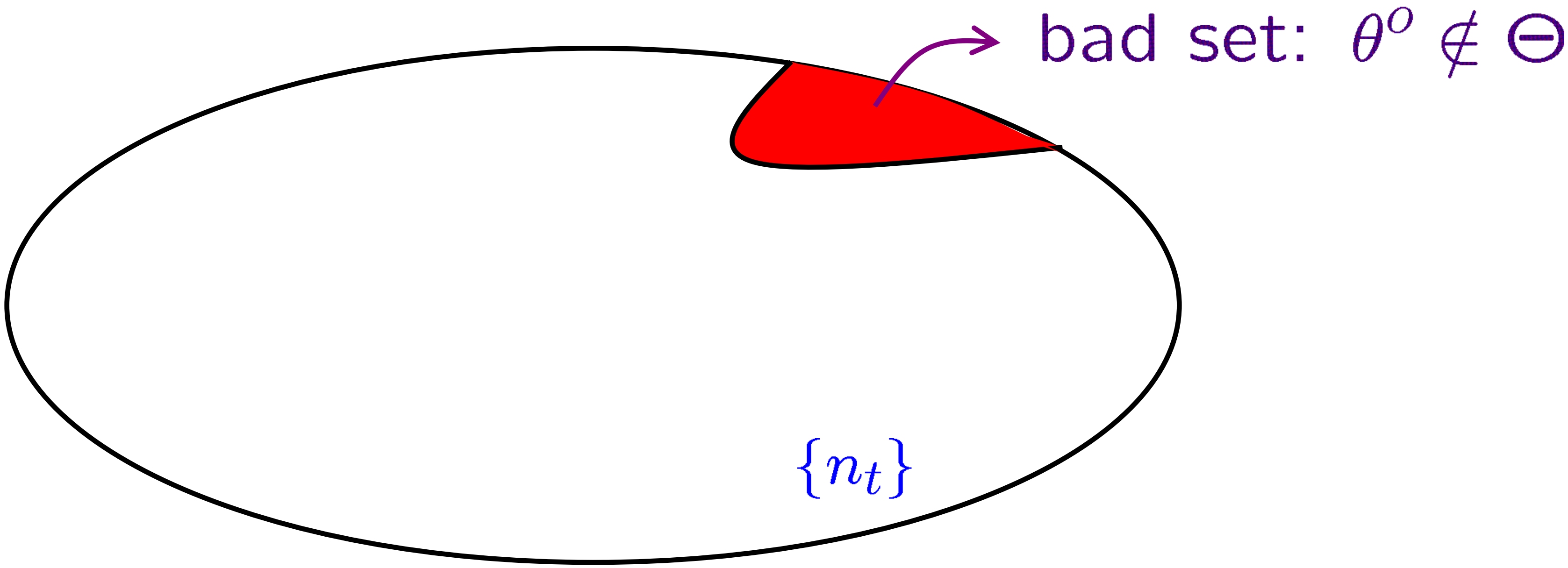
a probability is needed to tackle the challenge of being quantitative

remove the stringent deterministic assumptions



Claim: $\theta^o \in \Theta$ with probability 90%

remove the stringent deterministic assumptions



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strong probabilistic priors are used

Jan Willems: “where would the numerical values of this probability come from?”

move towards a distribution-free approach

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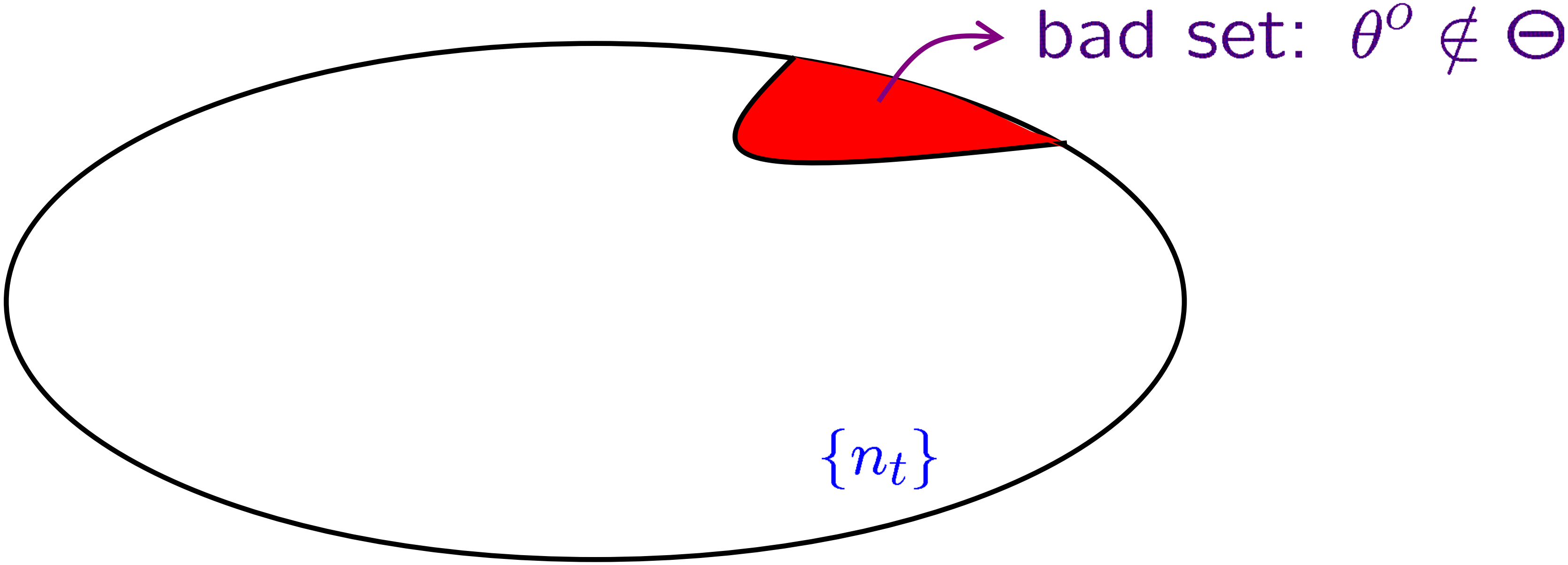

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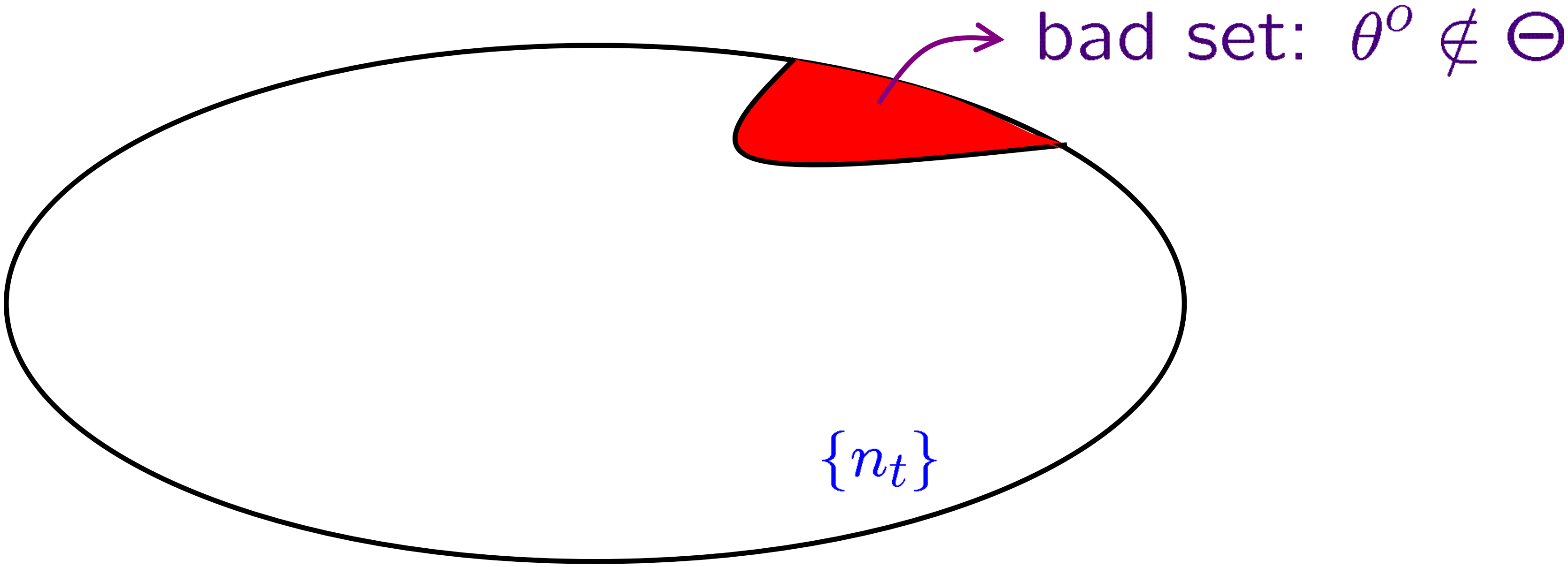

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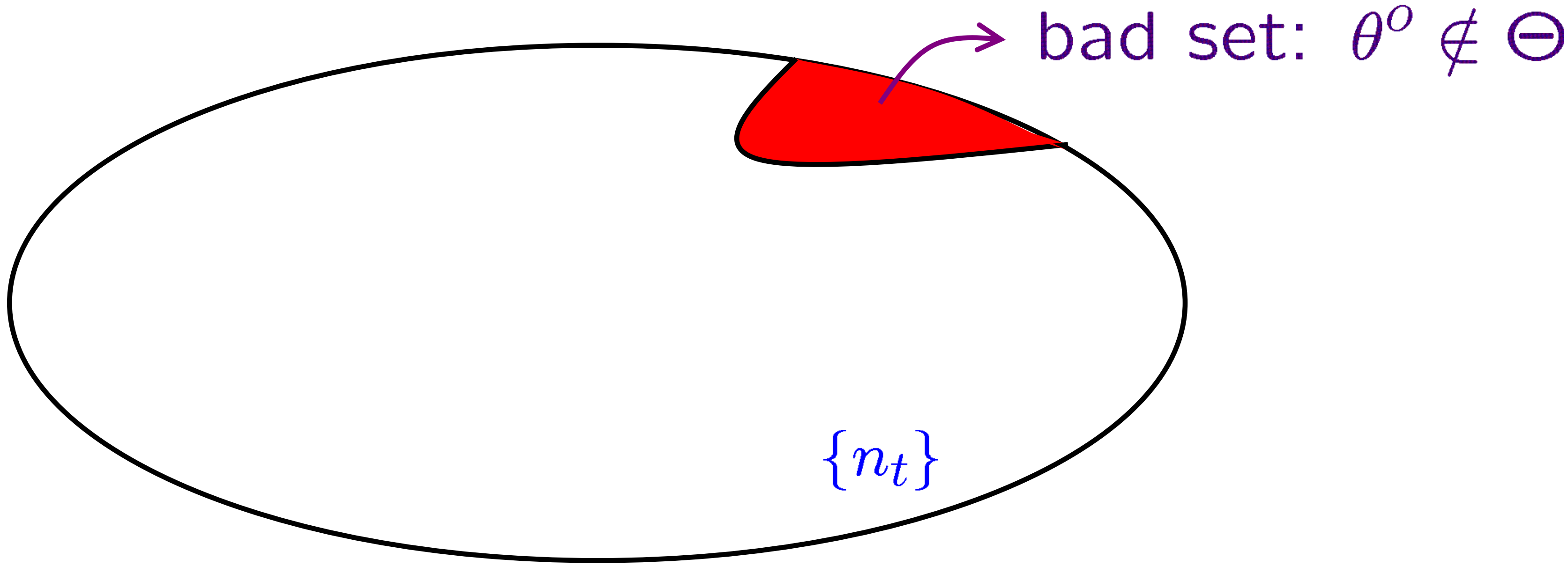

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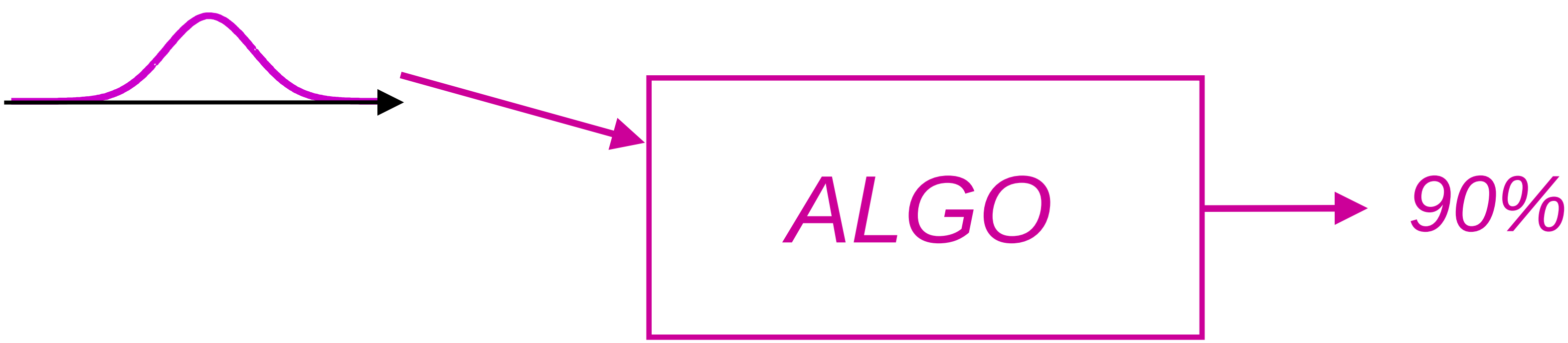

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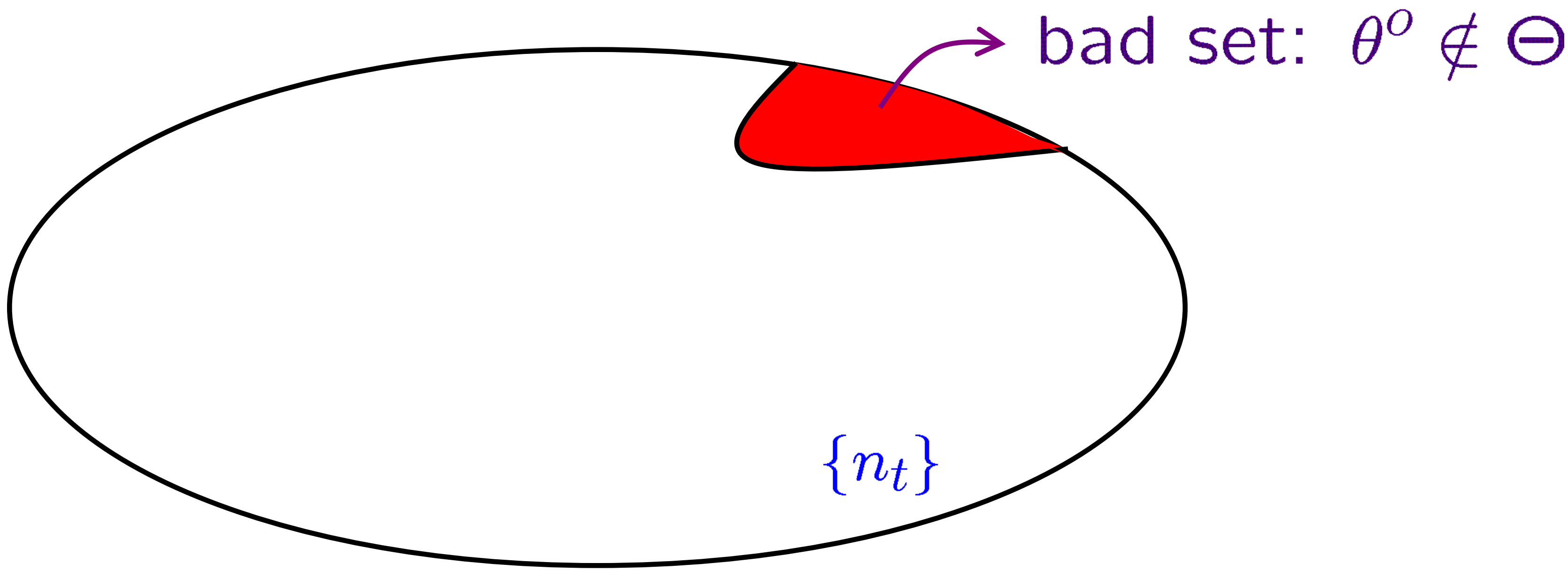

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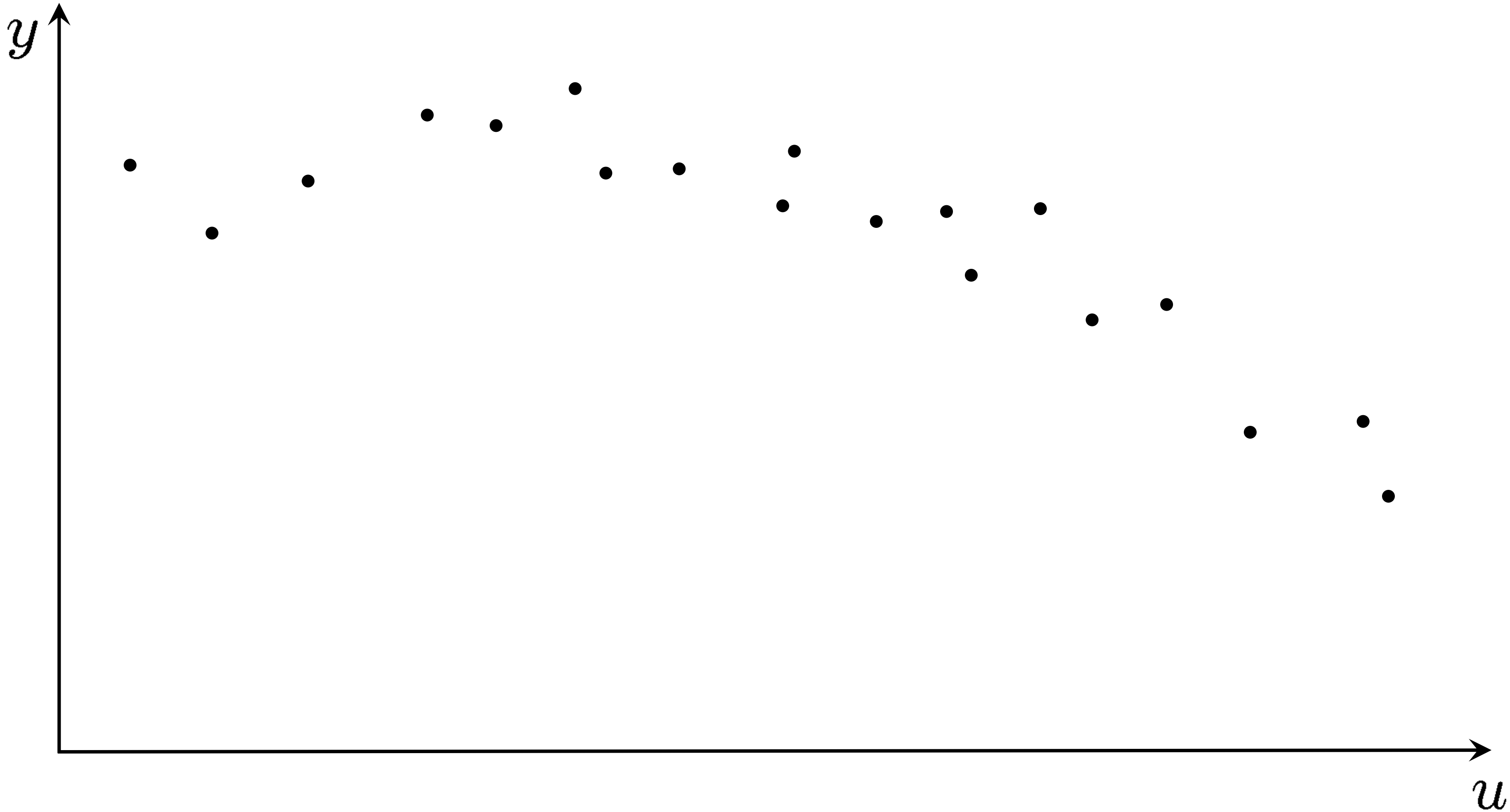
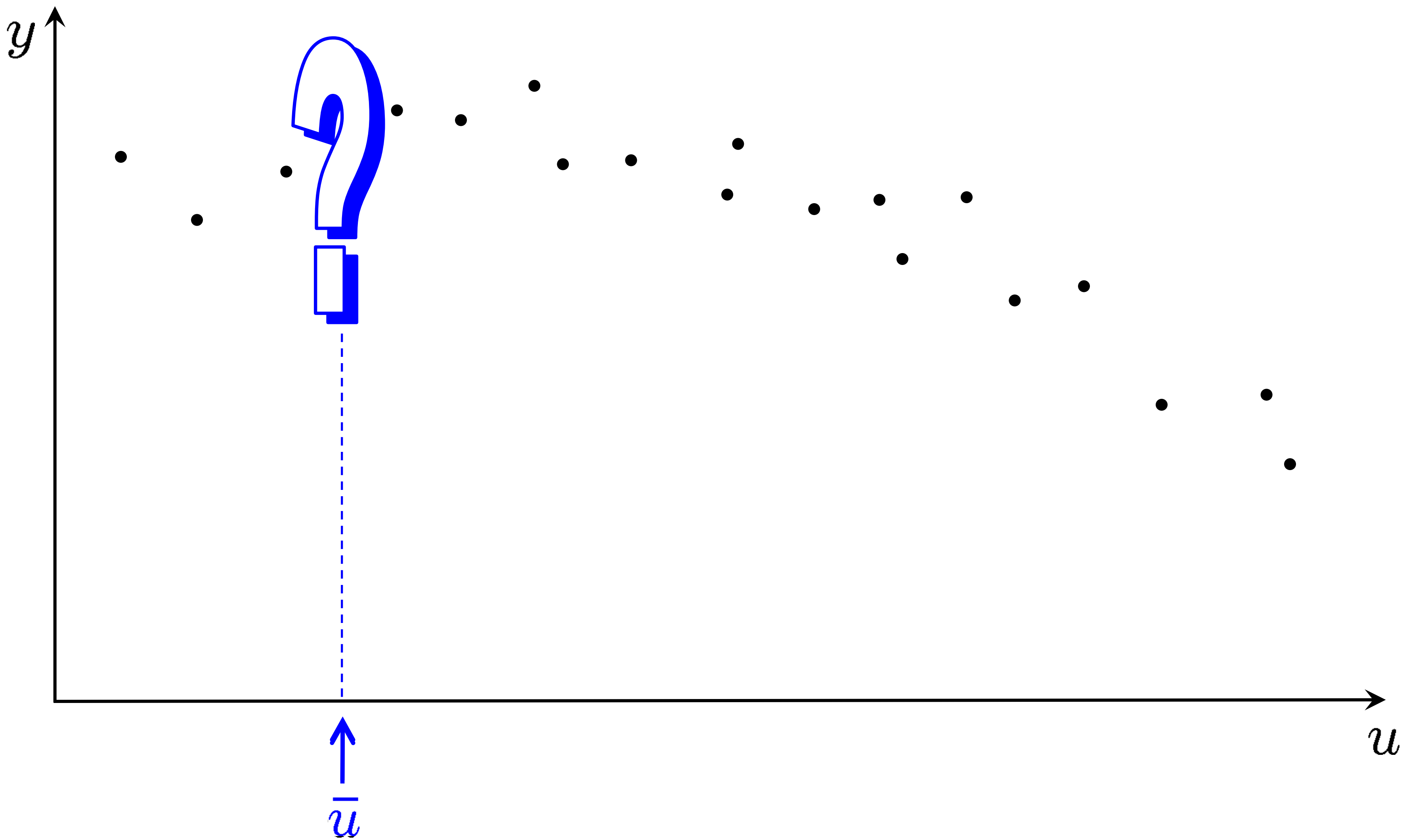
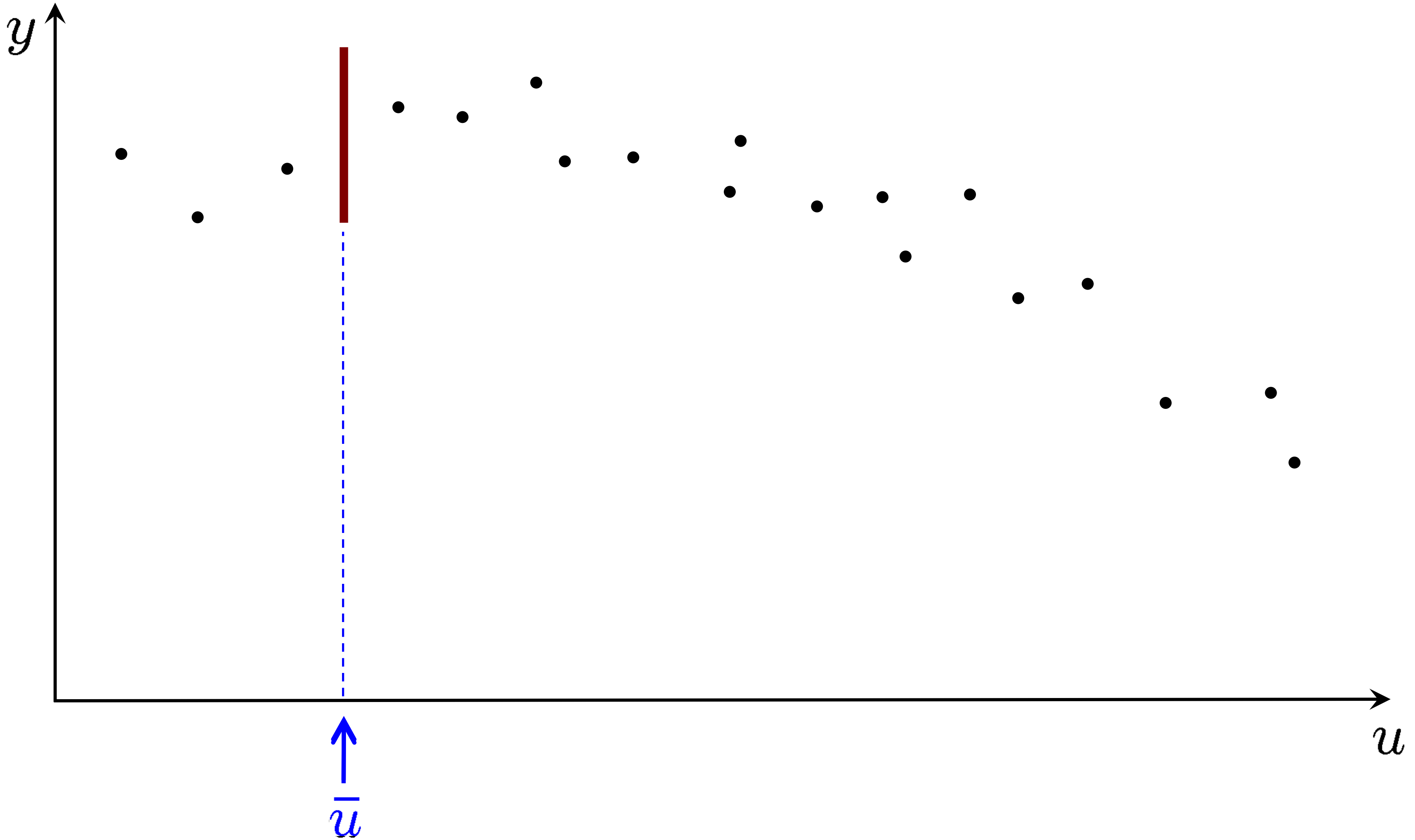


an example



an example



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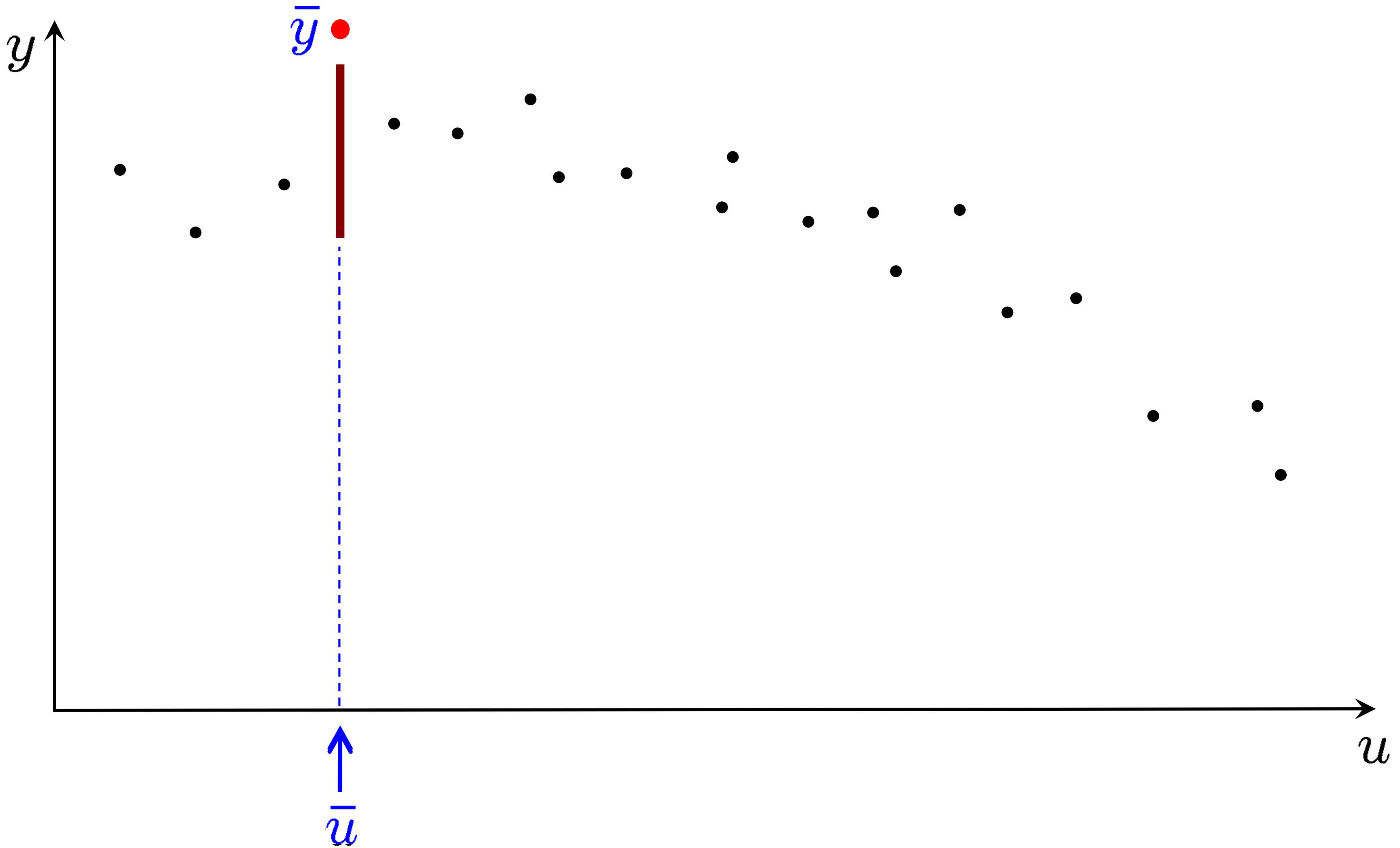

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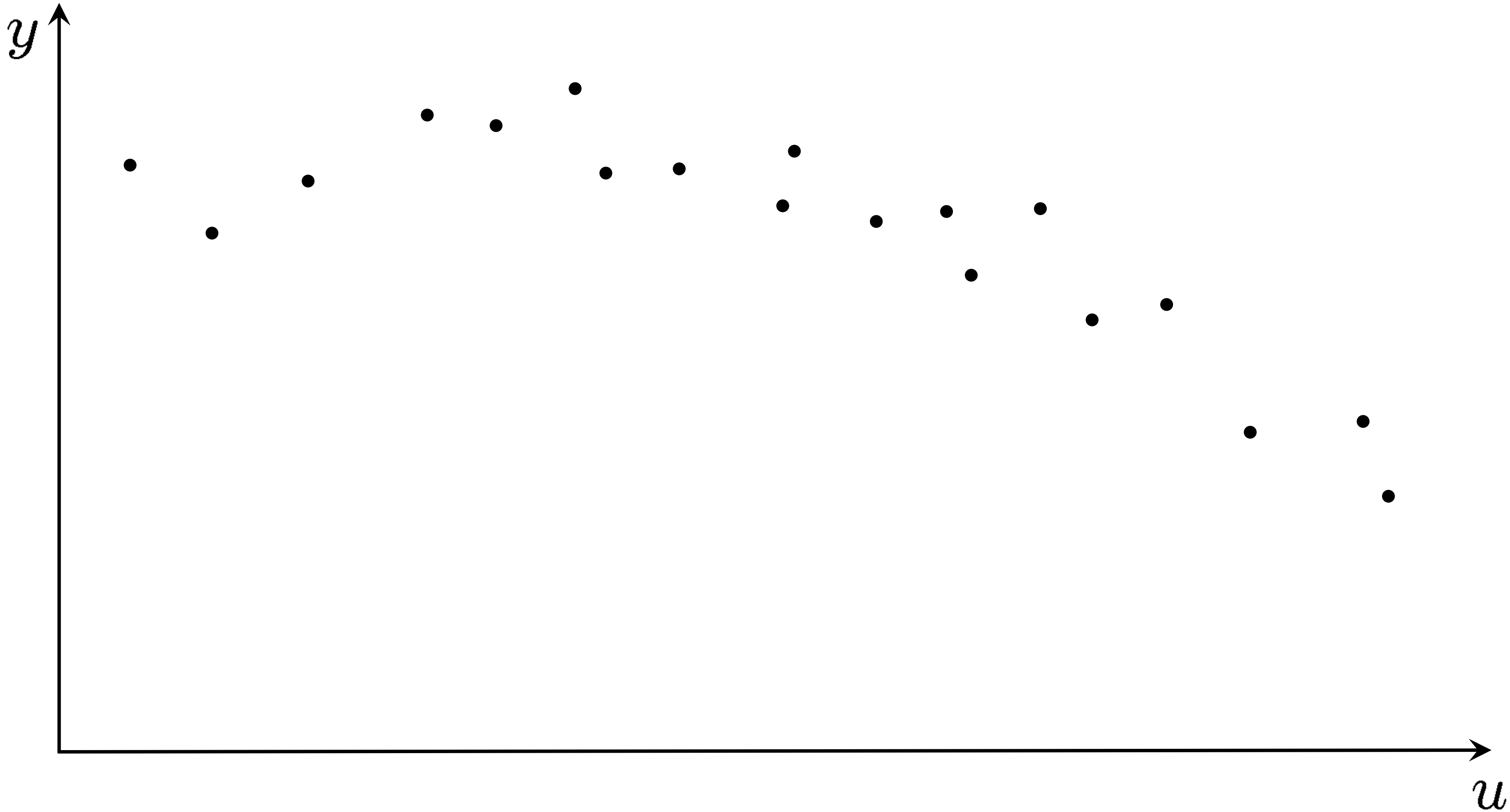
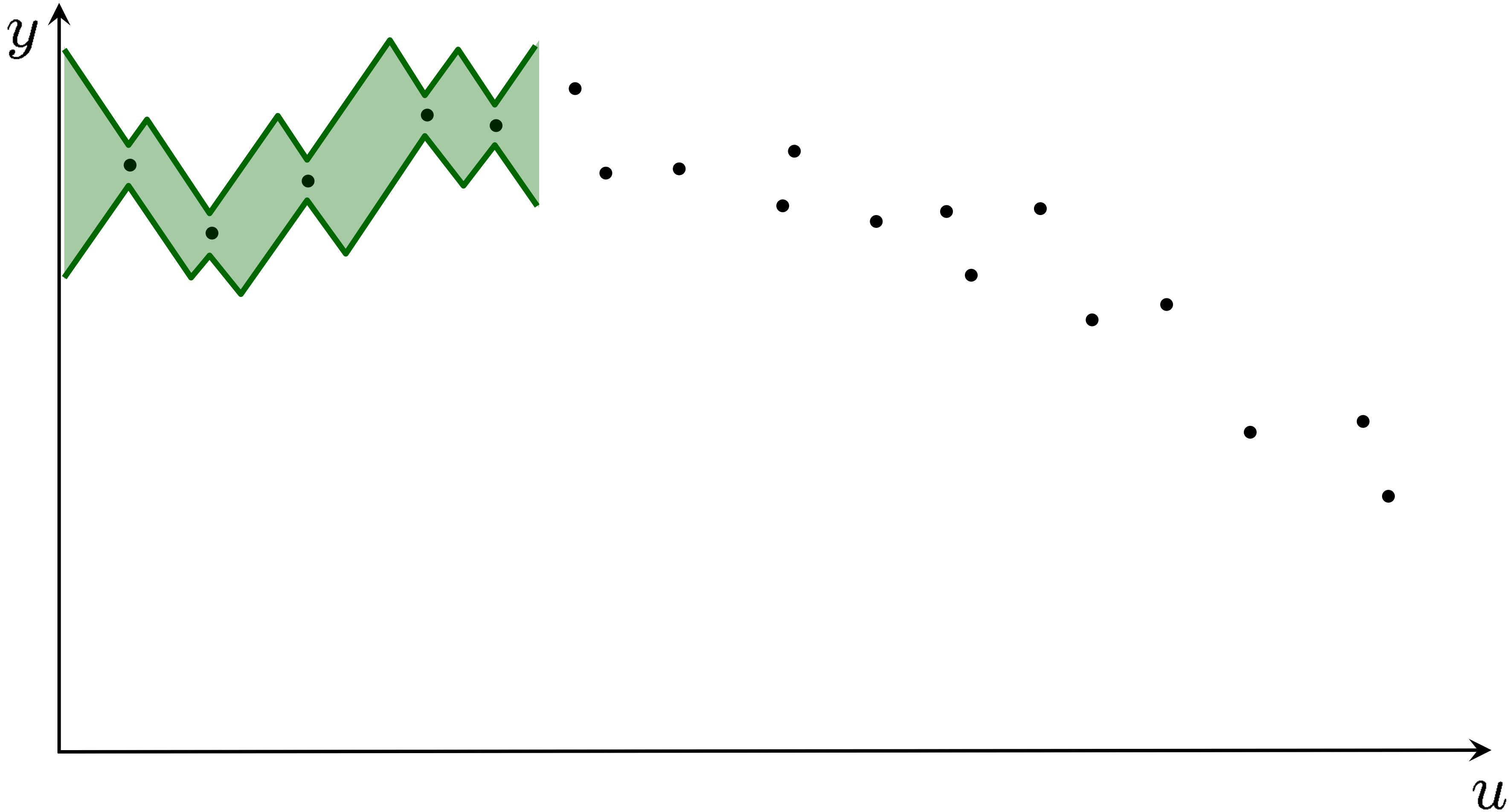


deterministic assumption: $y = f(u) + n$ $\quad \left|\dfrac{\partial f}{\partial u}\right| \leq 1$

$|n| \leq 0.5$

an example



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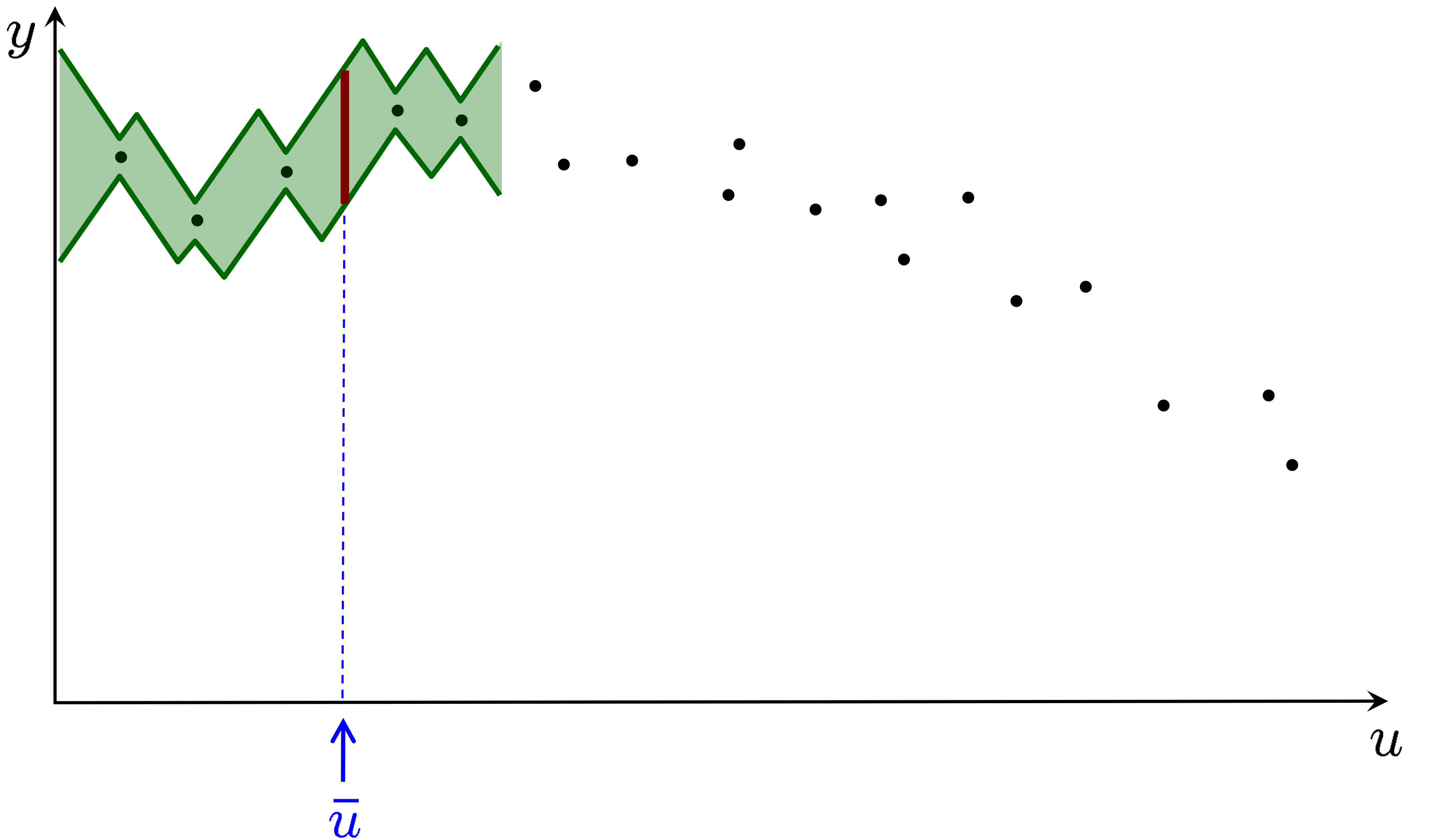
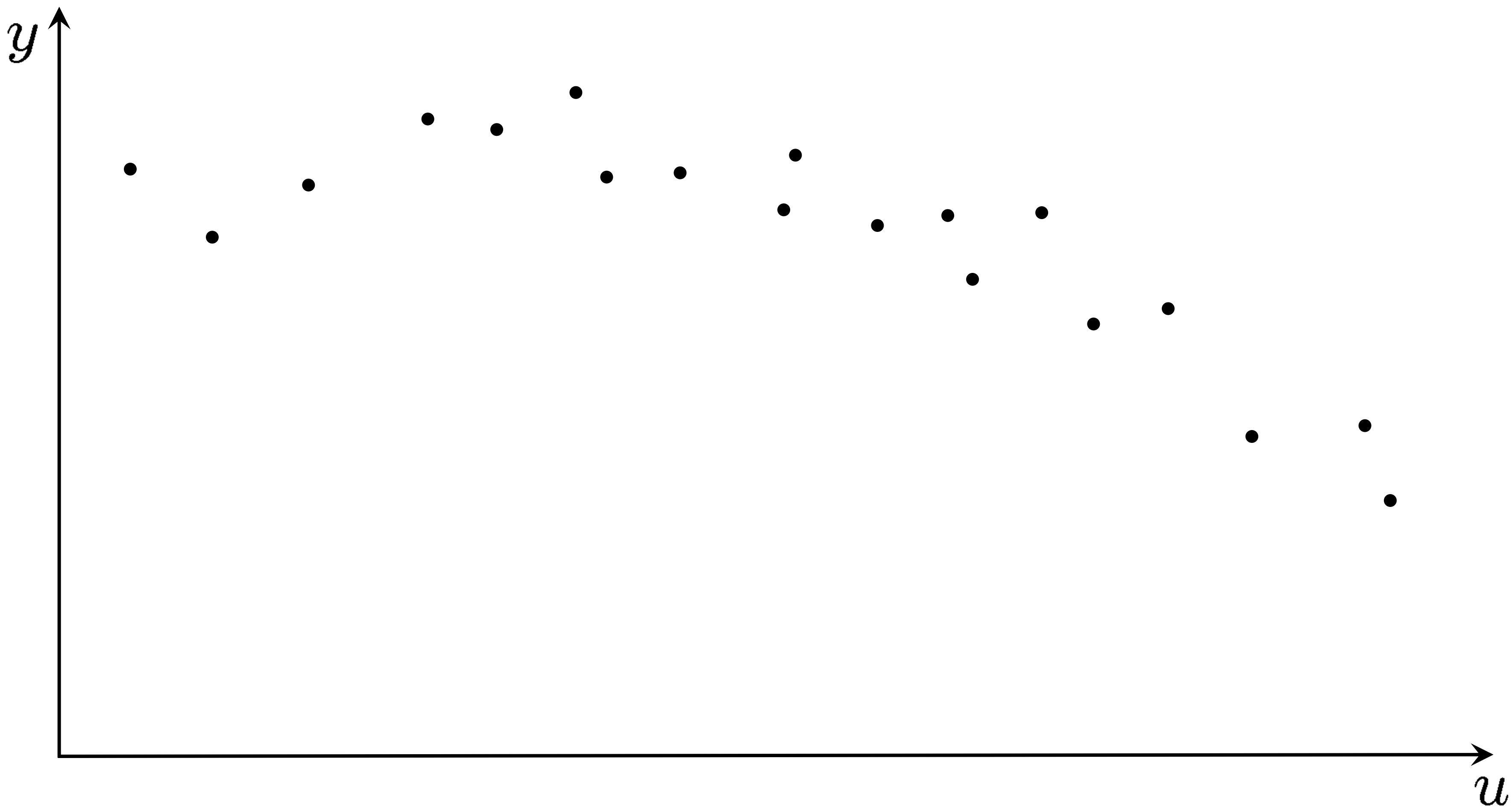


do we really believe in these priors?

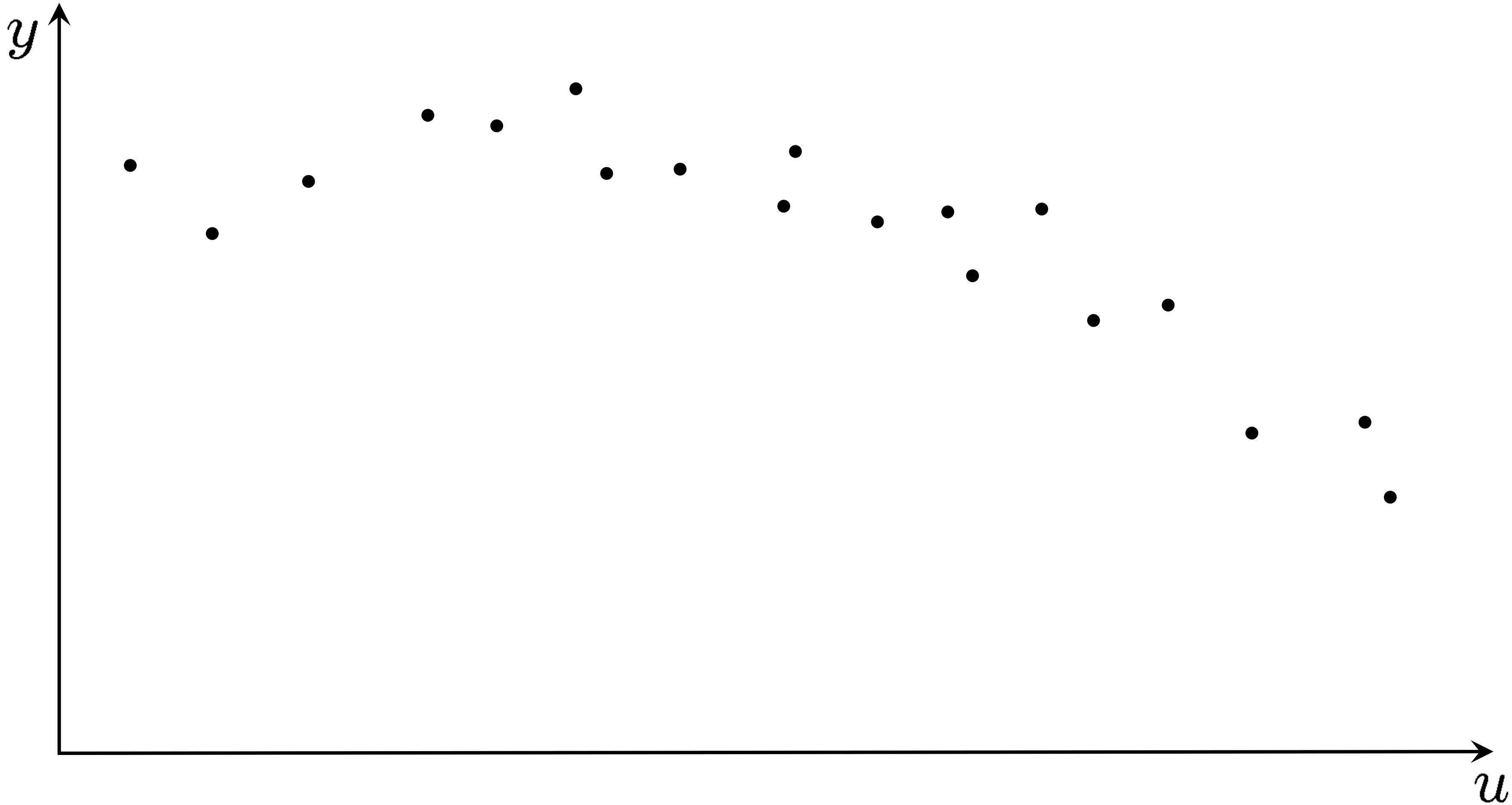
a paradigm shift

an example



no assumptions

an example



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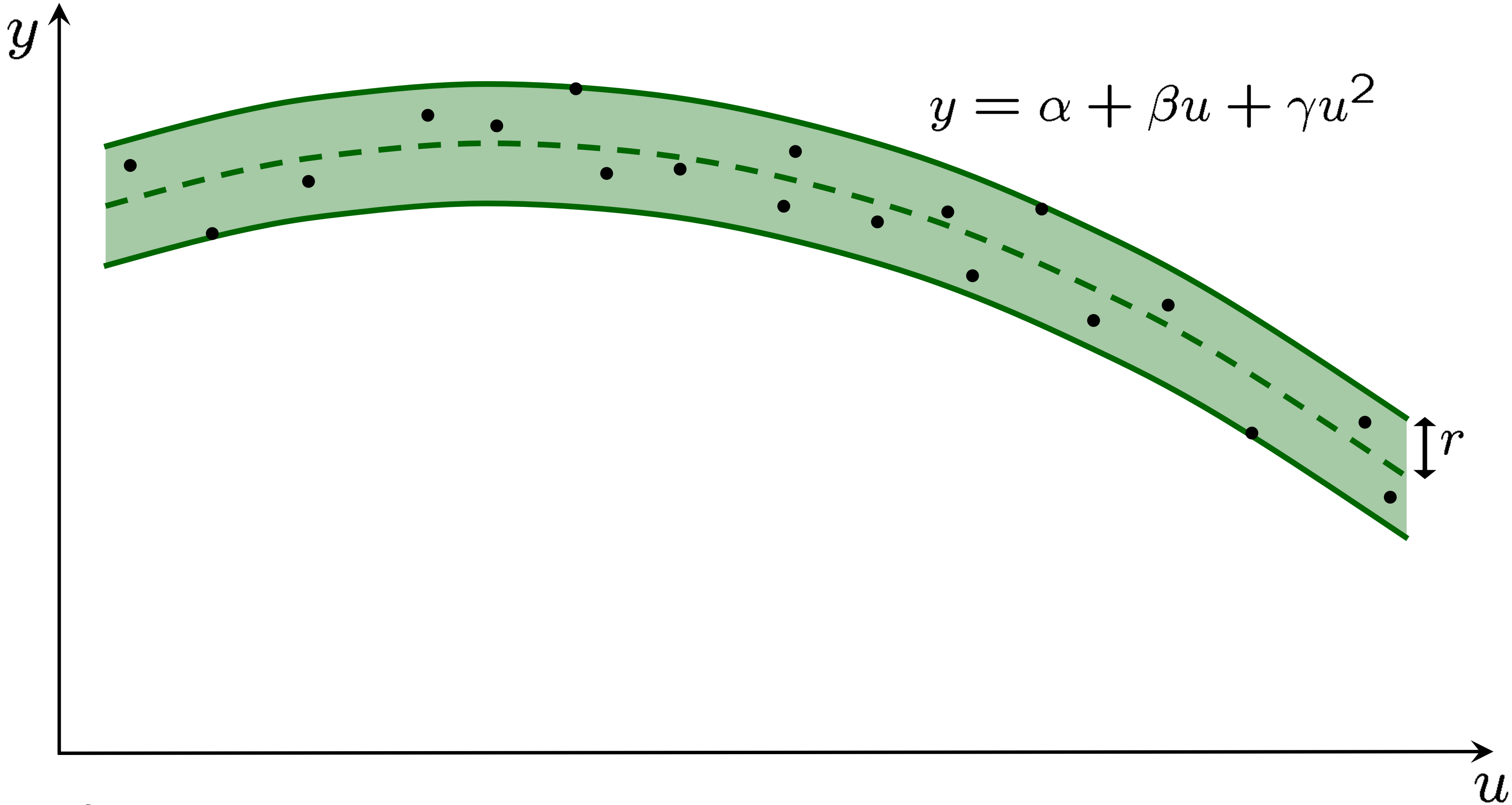
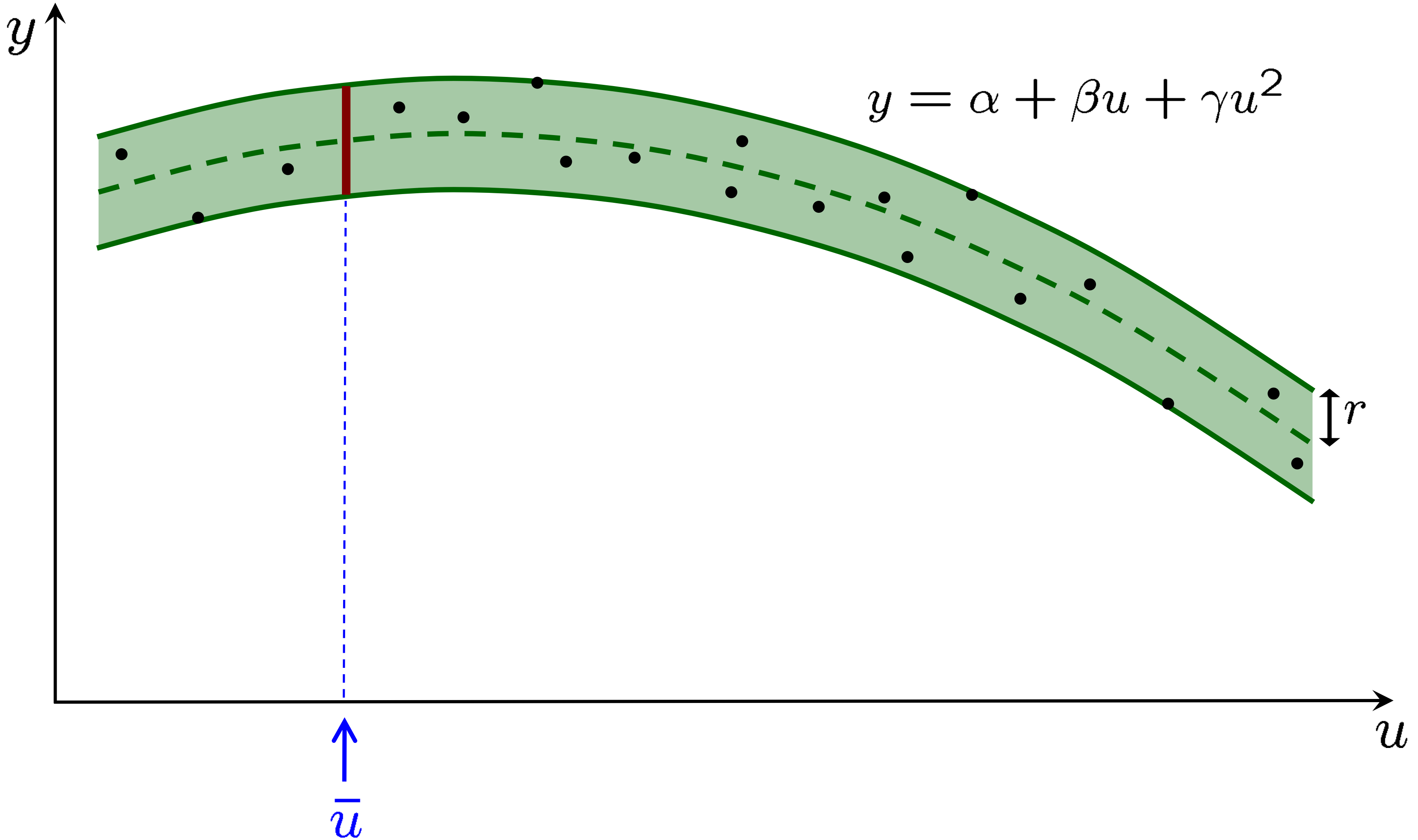


$$\min \quad r$$

$$\text{subject to:} \quad |y_i - [\alpha + \beta u_i + \gamma u_i^2]| \leq r, \quad i = 1, \ldots, 19$$

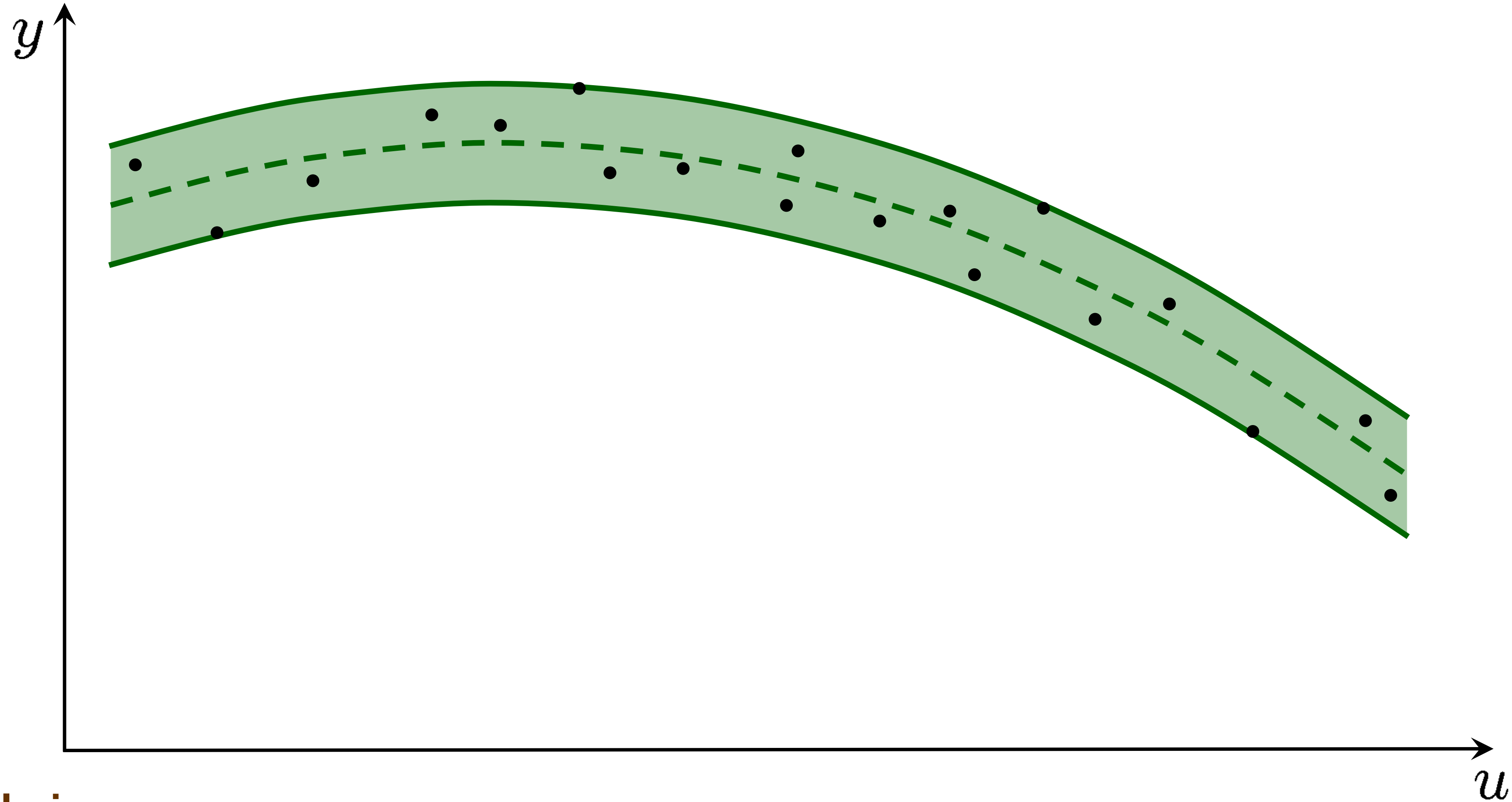
[Campi, Calafiore, Garatti, Automatica, 2009]

an example



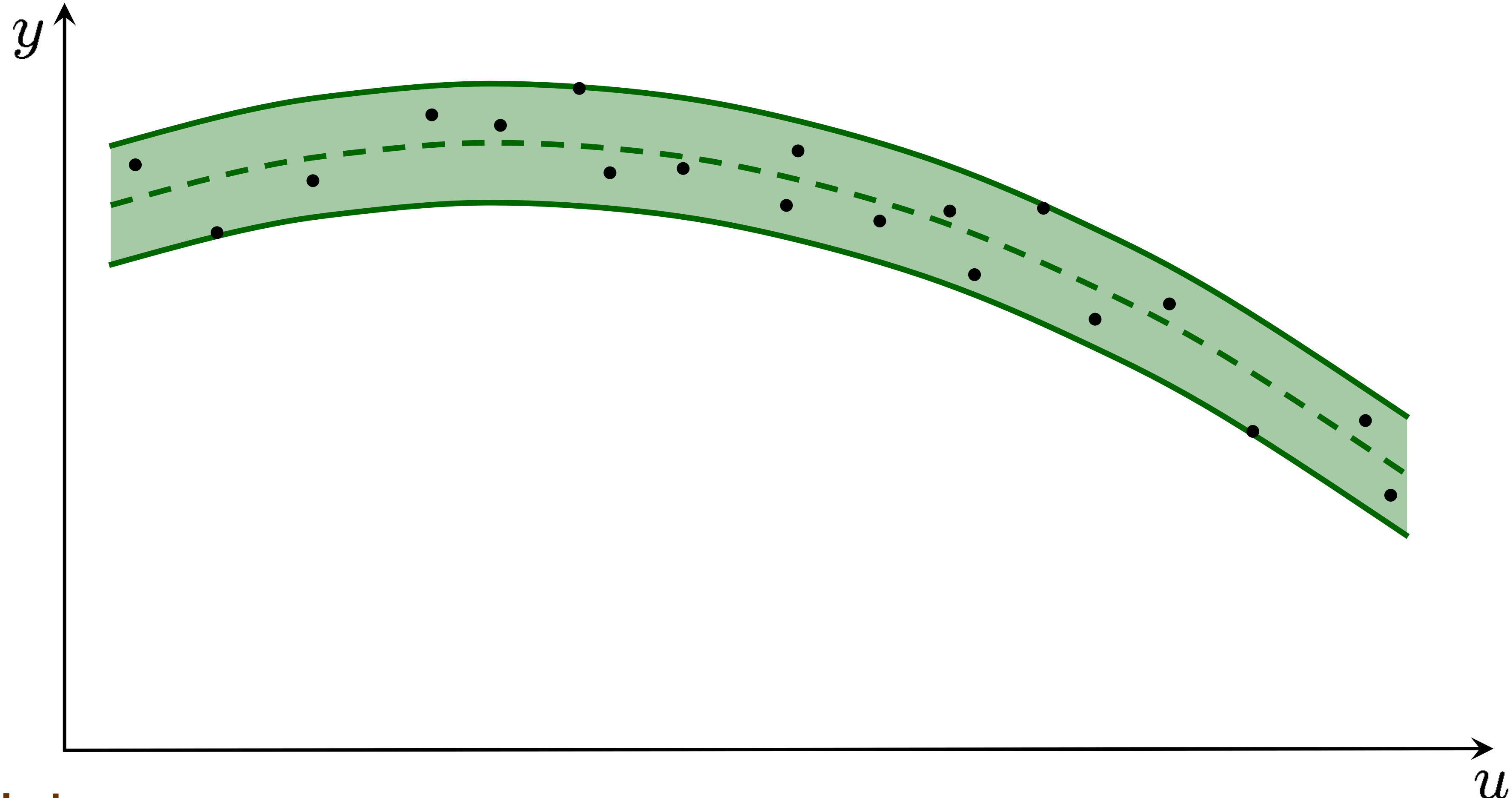
[Campi, Calafiore, Garatti, Automatica, 2009]

an example



Claim: The prediction is correct with exact probability 80%, irrespective of the probability with which data are generated

an example



Claim: The prediction is correct with exact probability 80%, irrespective of the probability with which data are generated

totally distribution-free!

Theorem [Campi, Calafiore, Garatti, Automatica, 2009]

k = degrees of freedom of centerline
N = # of data points

The layer predicts correctly with exact probability p if

$$N = \frac{k+p}{1-p}$$

Theorem [Campi, Calafiore, Garatti, Automatica, 2009]

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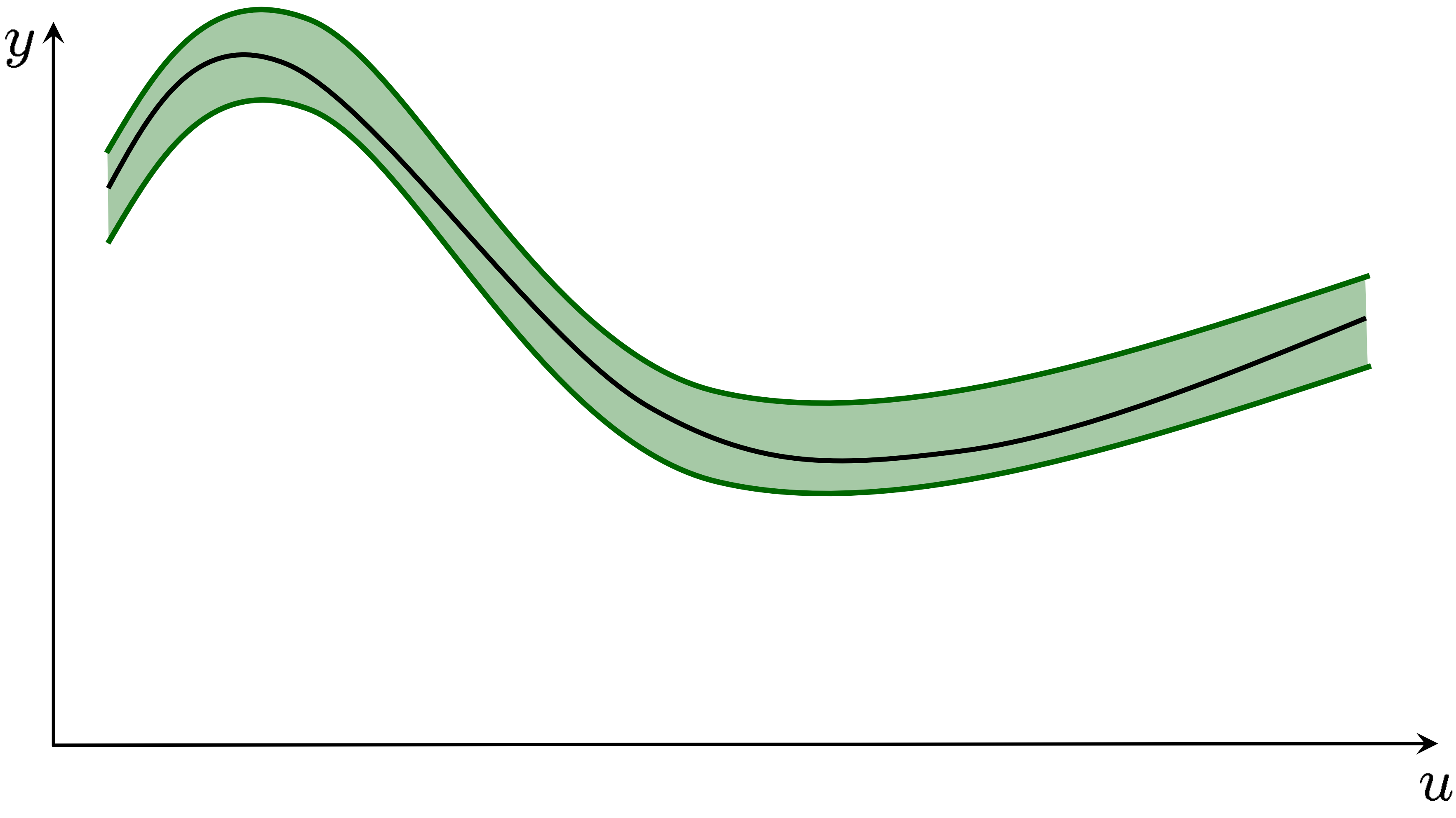
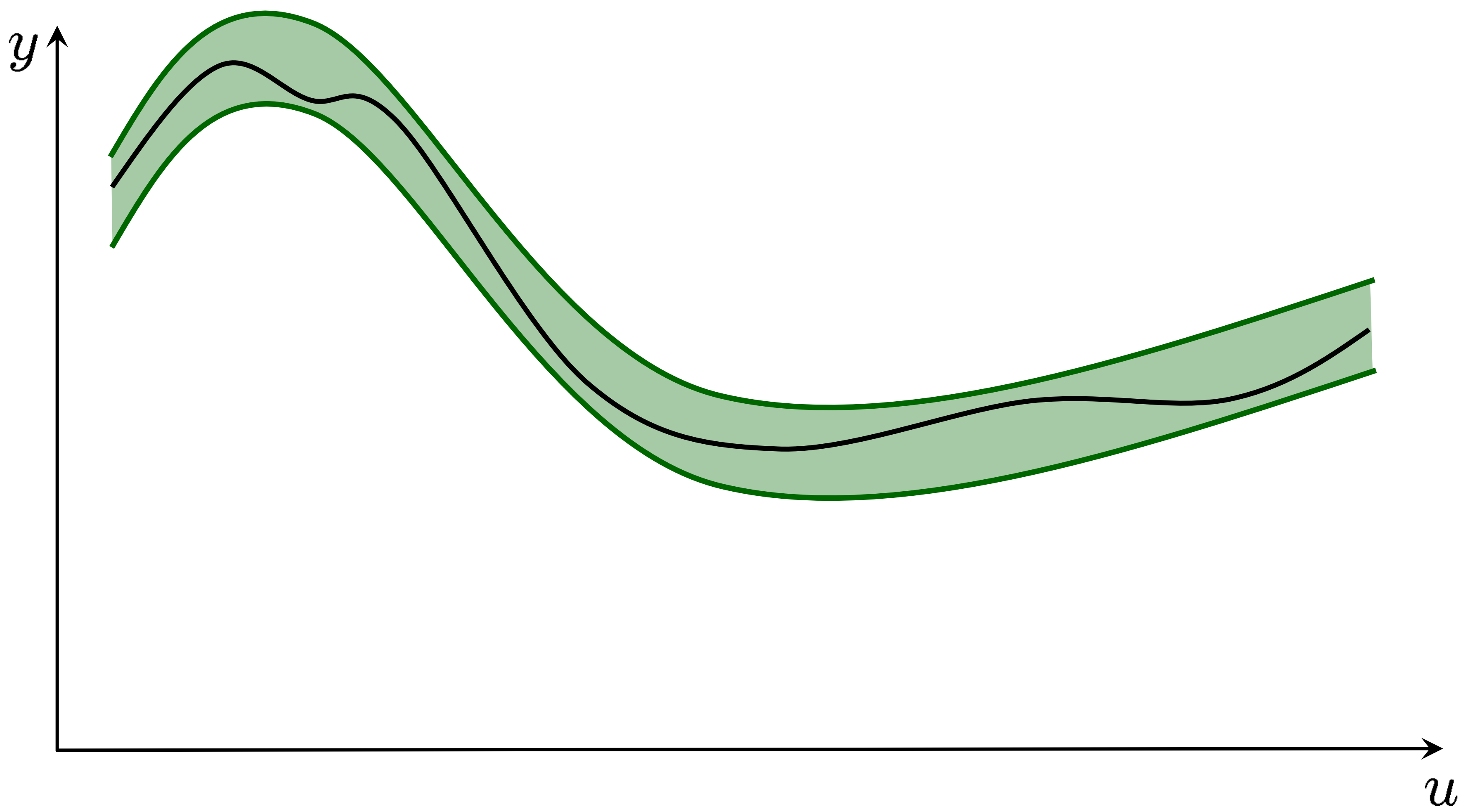
$p = 80\%$ $\Rightarrow$ $\frac{k+p}{1-p} = \frac{3+0.8}{1-0.8} = 19$

why is this possible?



y
u



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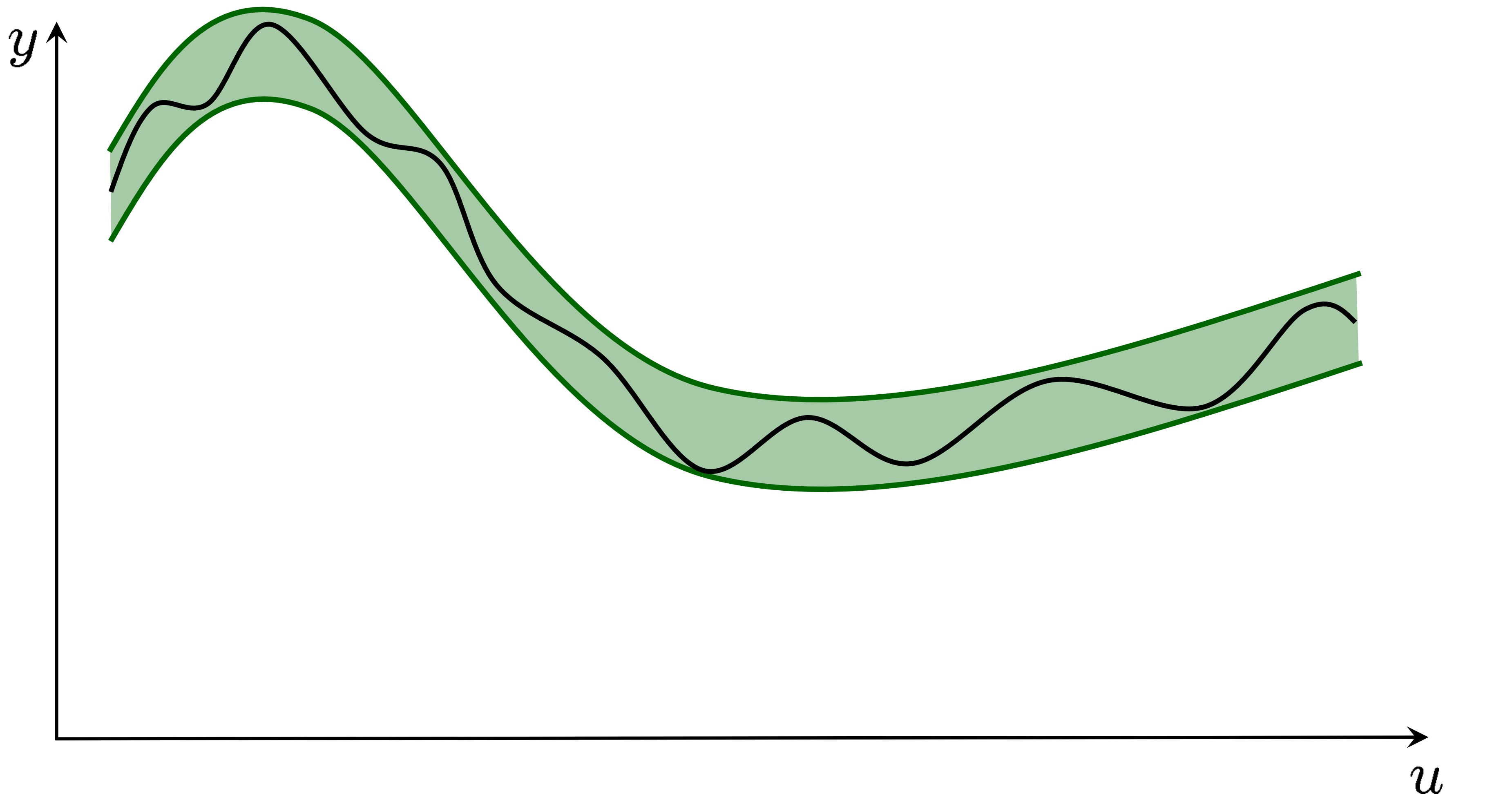

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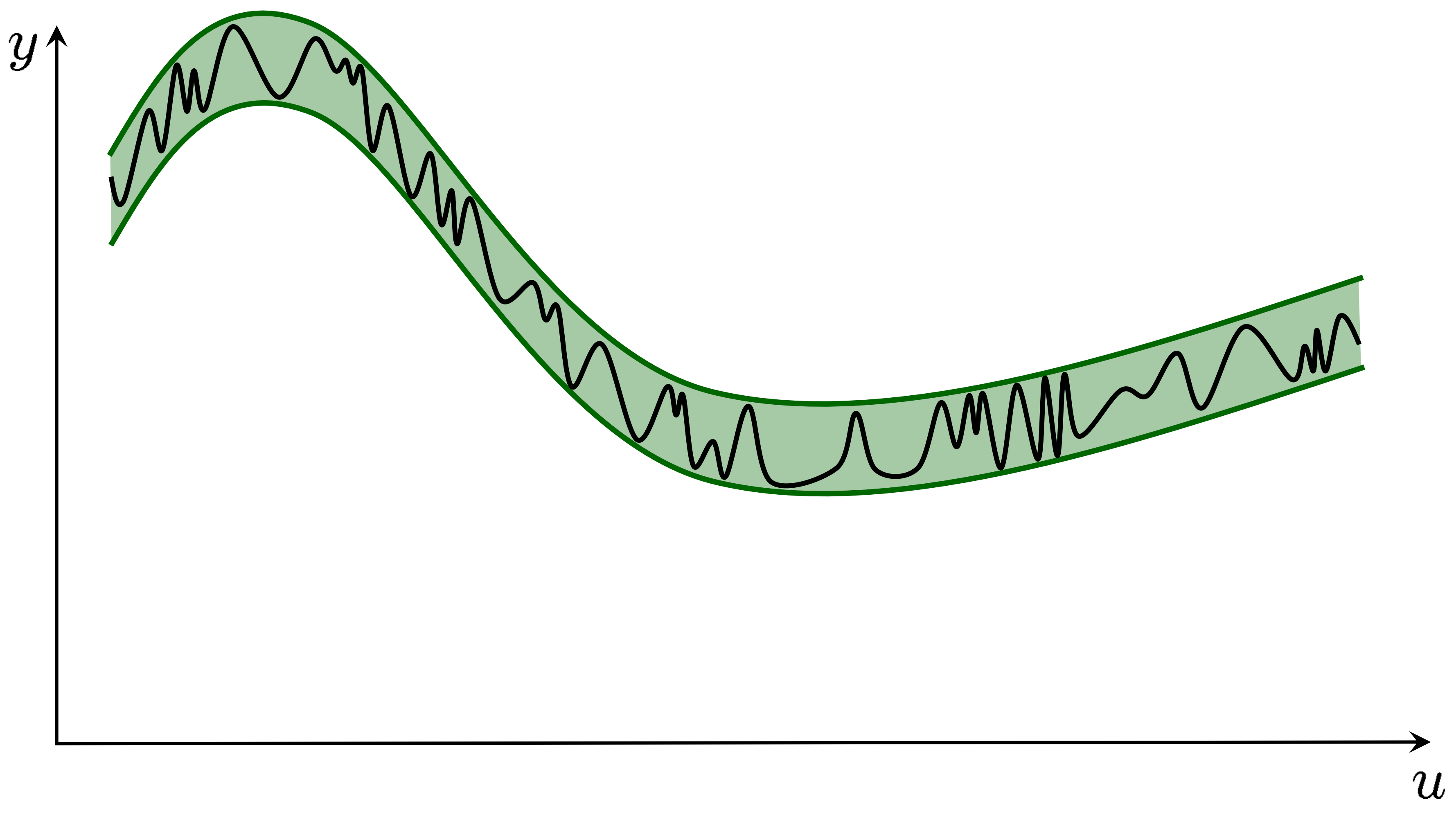
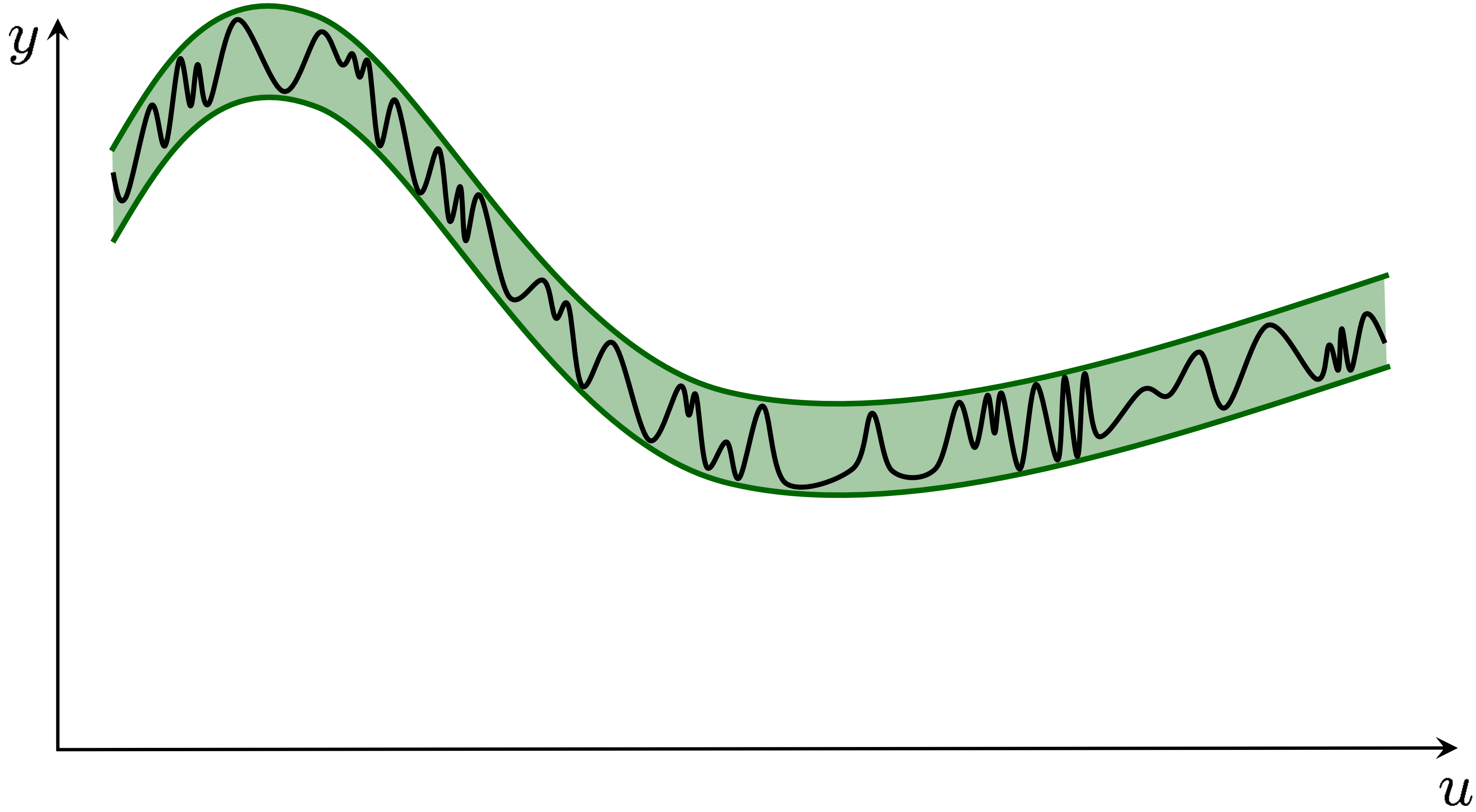
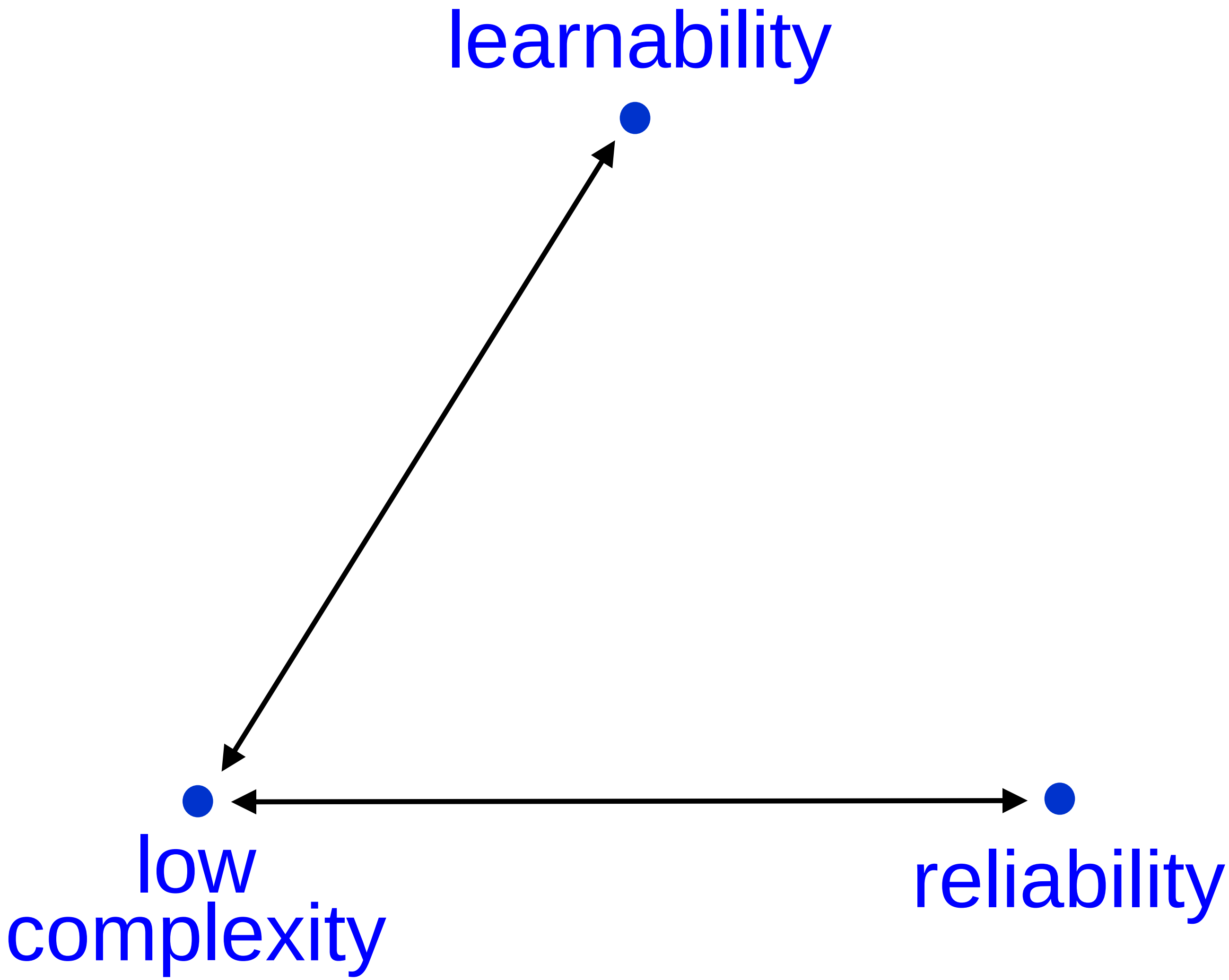


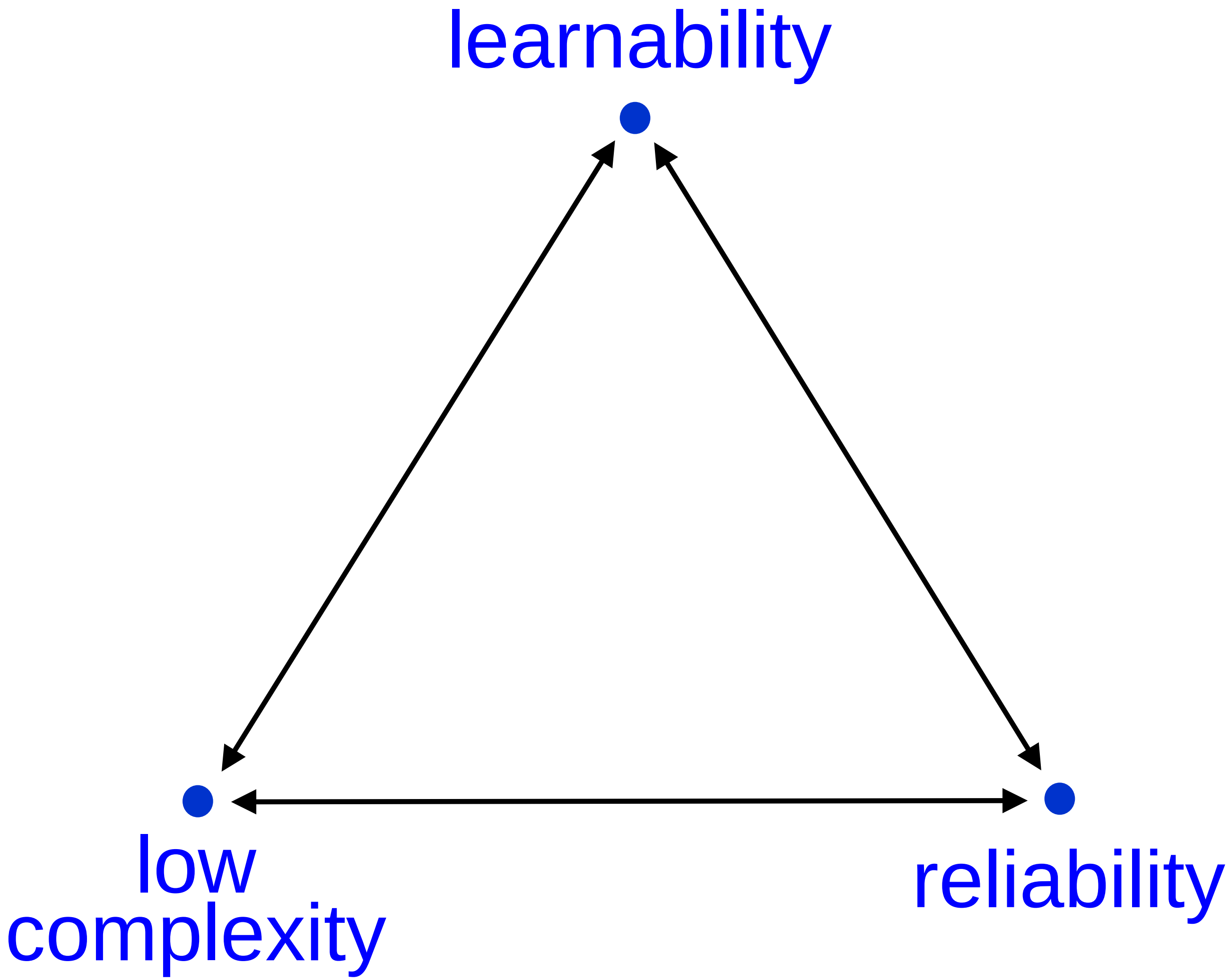
y
u

a simple predictor can be reliable for complex data generation mechanisms




low
complexity
reliability


learnability
low
complexity
reliability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what is the role of priors?

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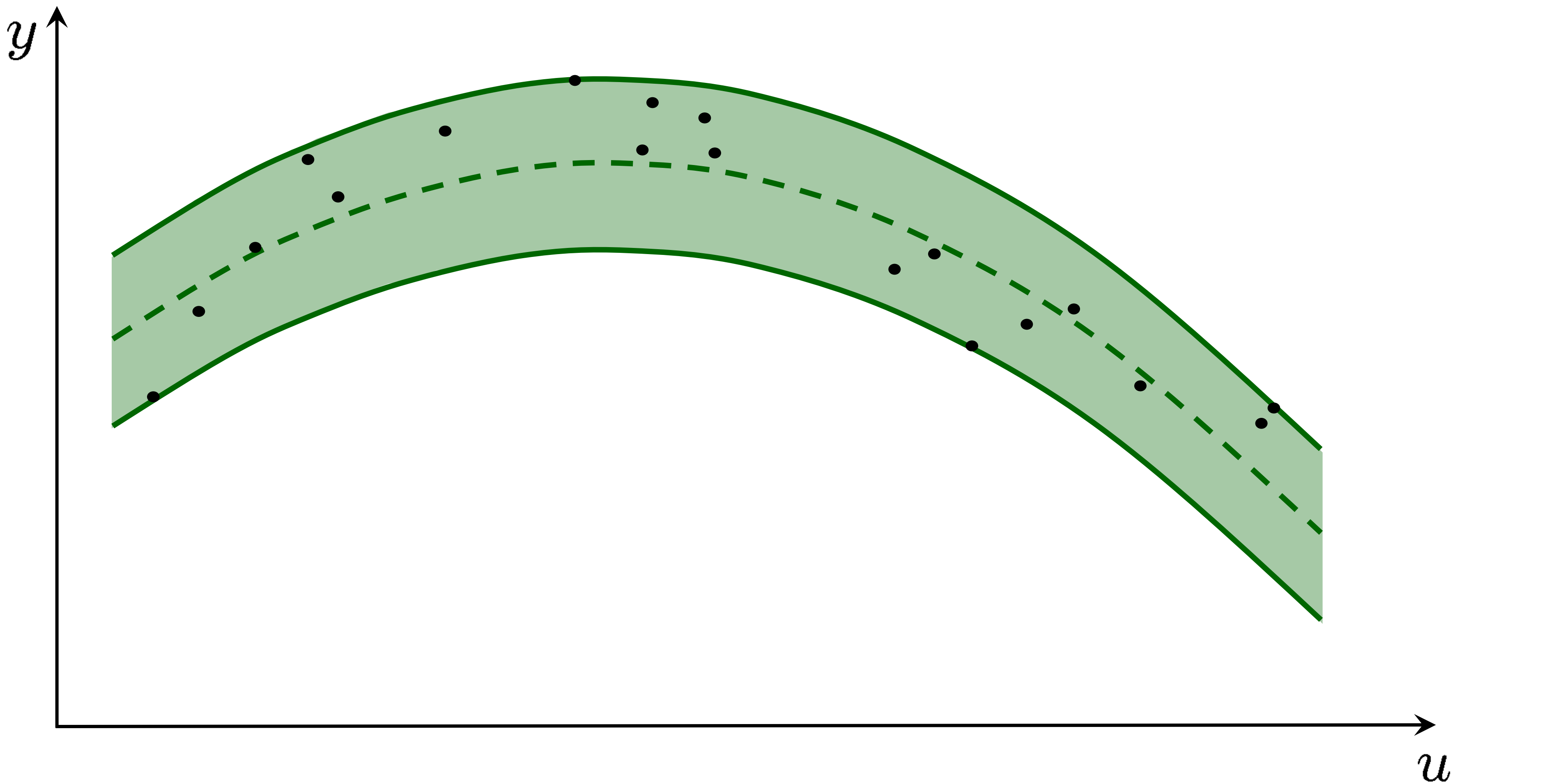

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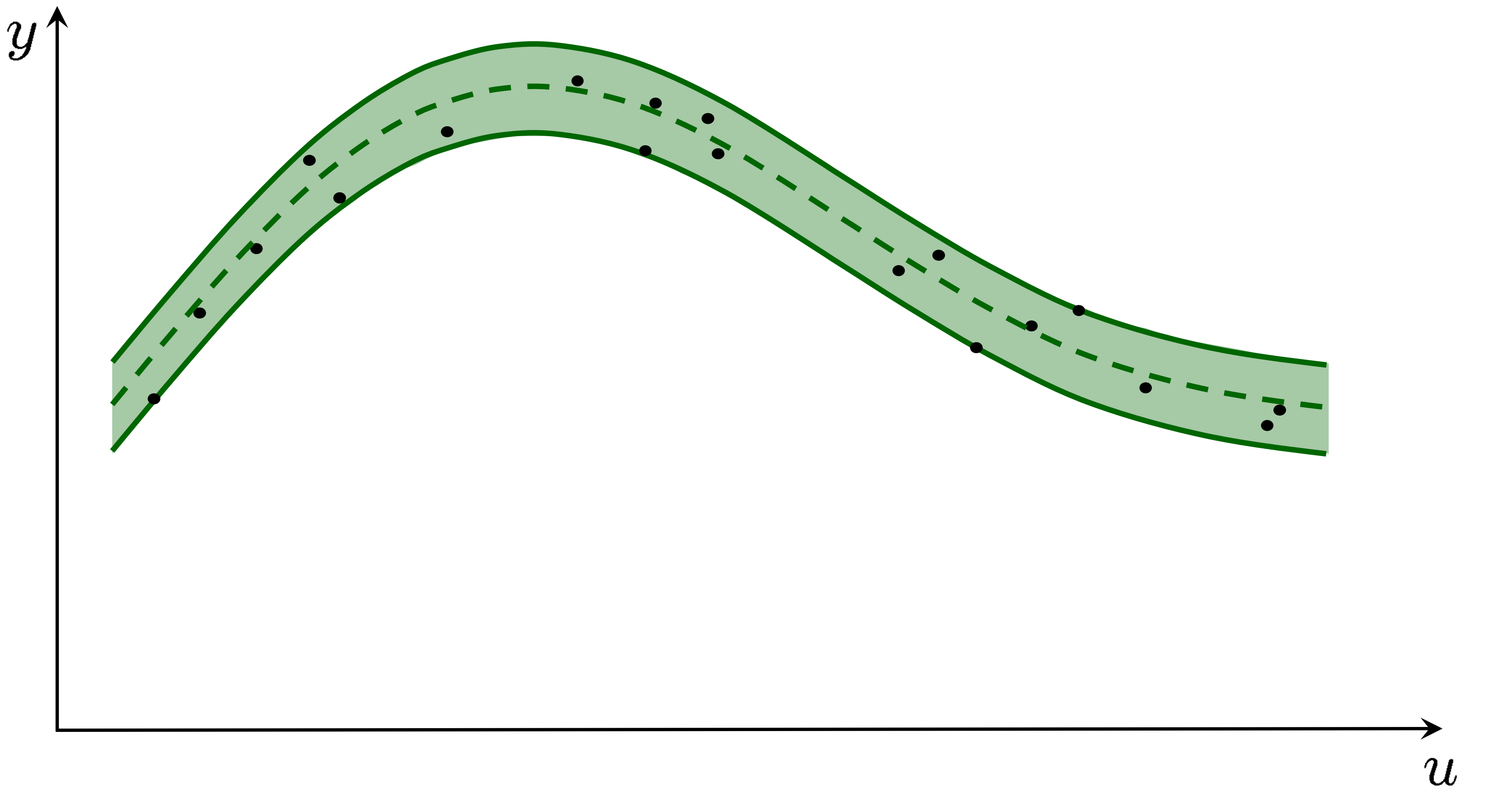

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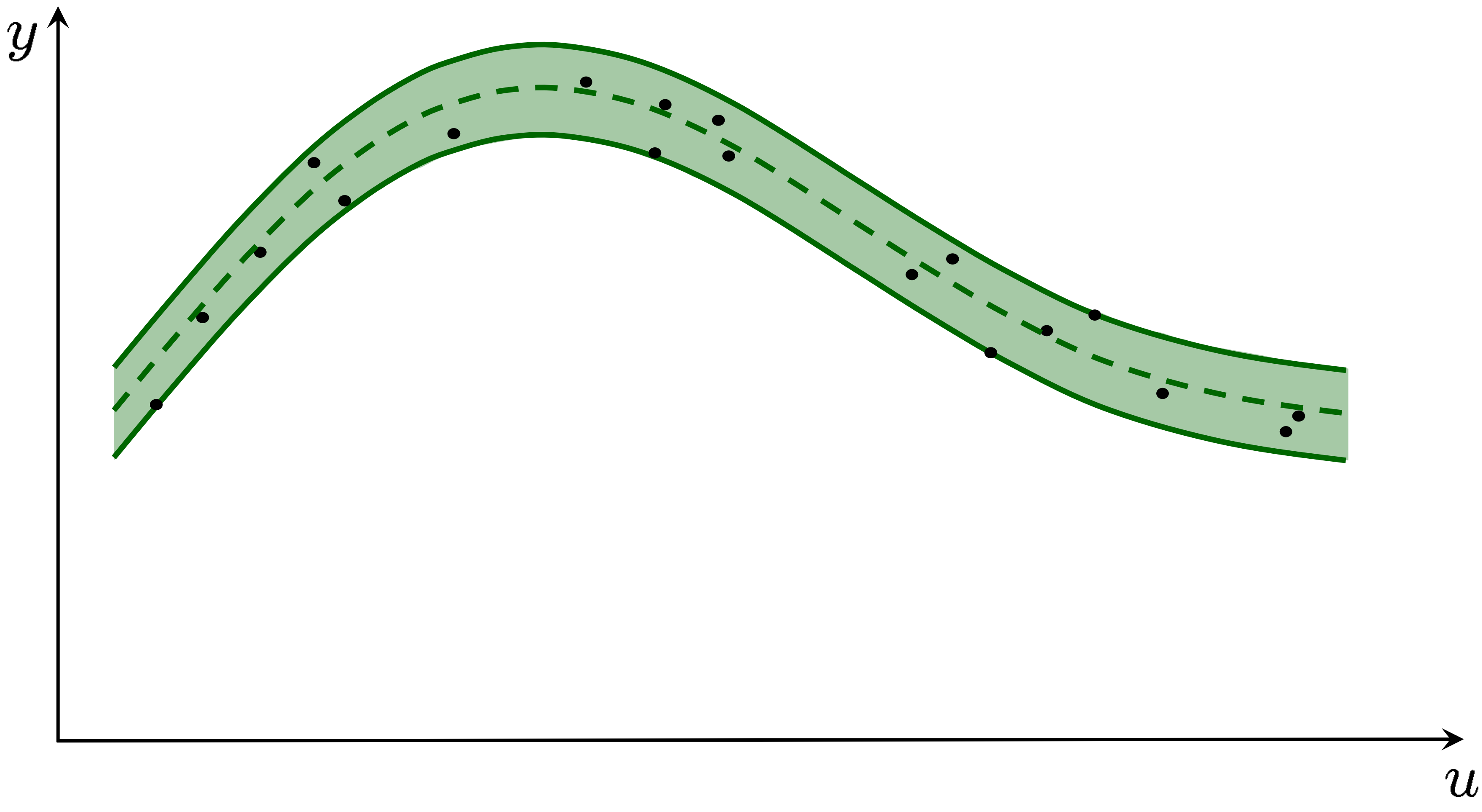

“thickness”

an interesting fact:

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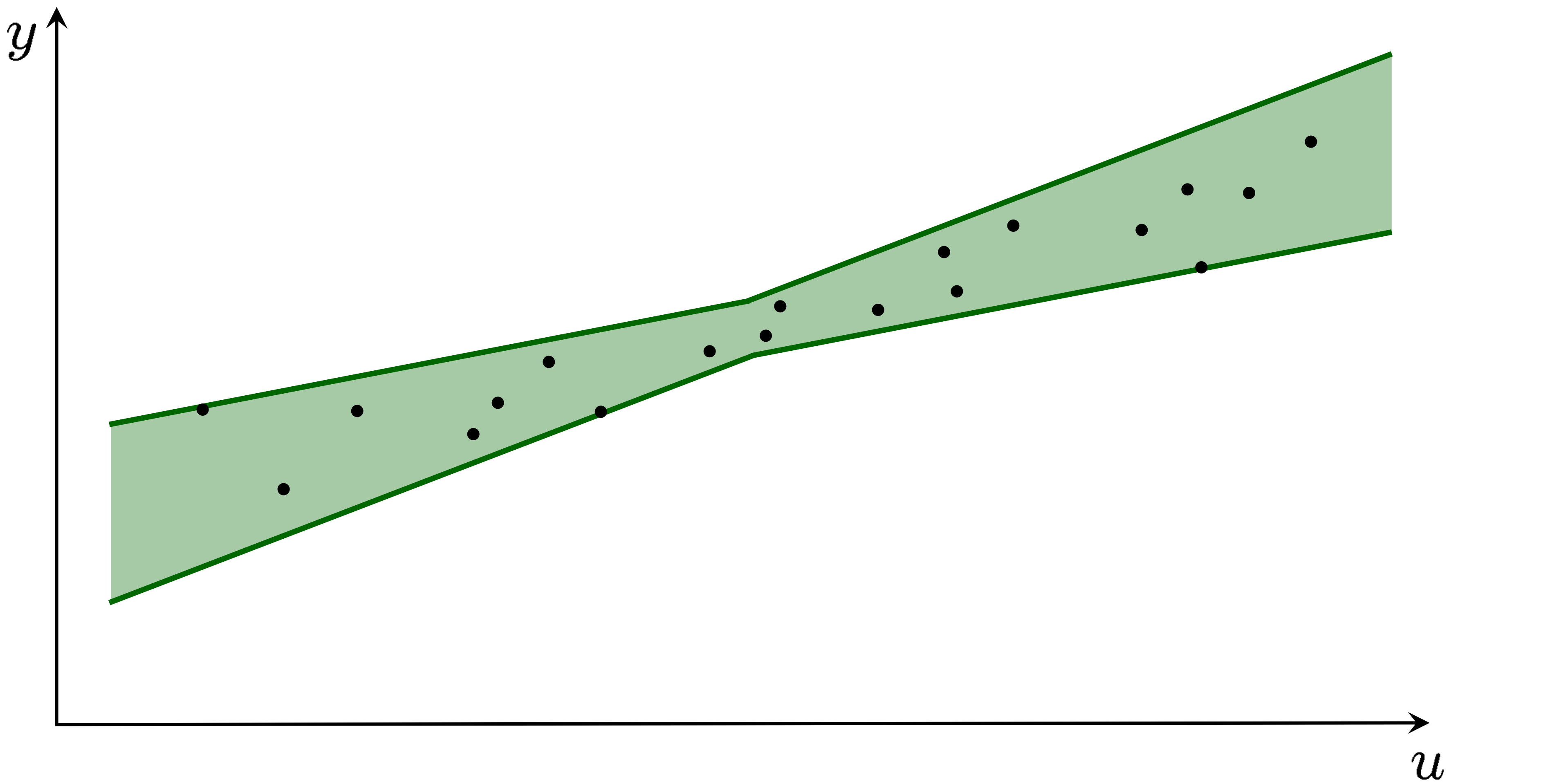
  thickness(model).

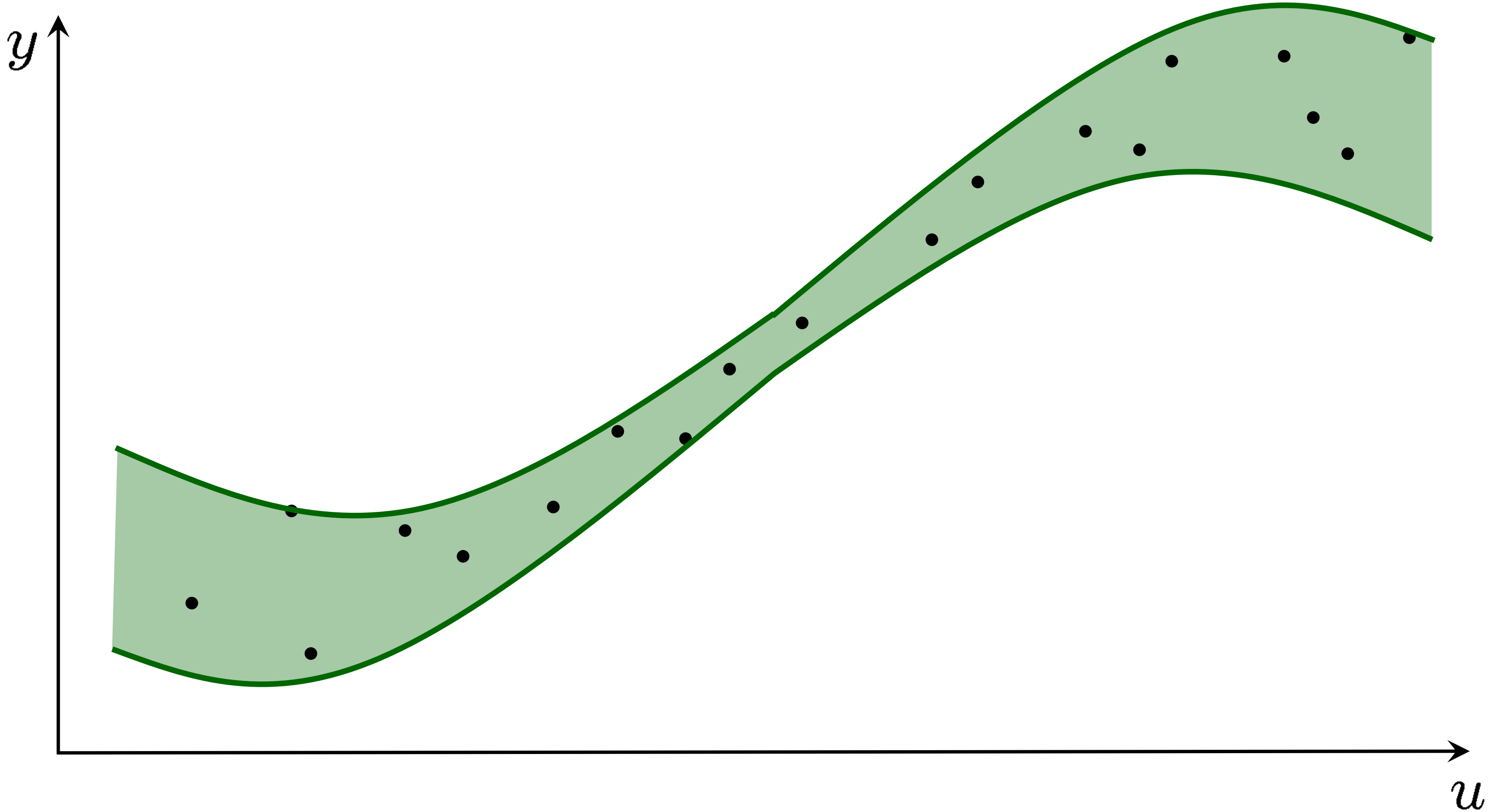
So: thickness can be inspected, while reliability has to be guaranteed by a theory valid under general assumptions.

algorithms for ...



[Campi, Calafiore, Garatti, Automatica, 2009]

algorithms for …



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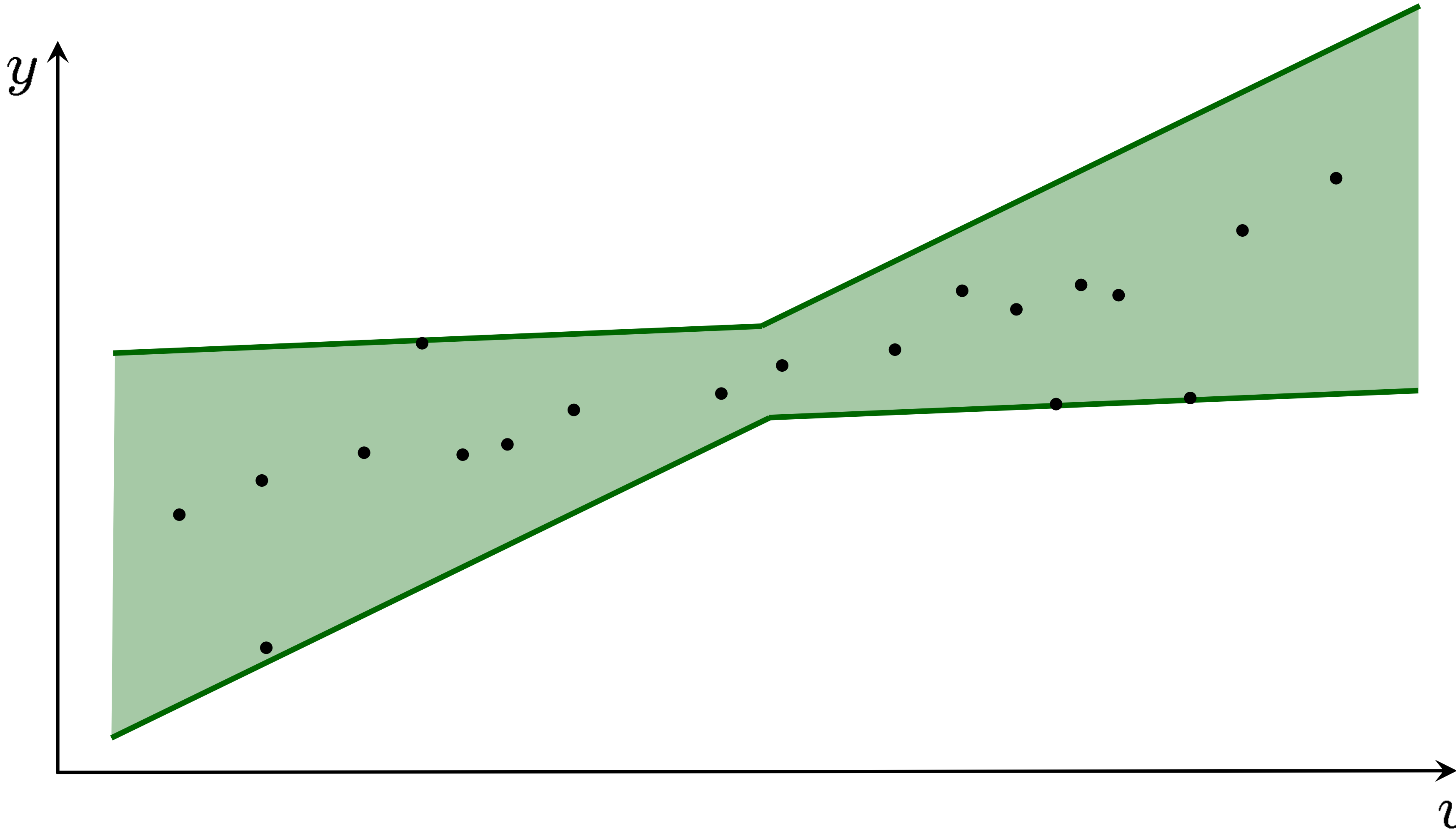

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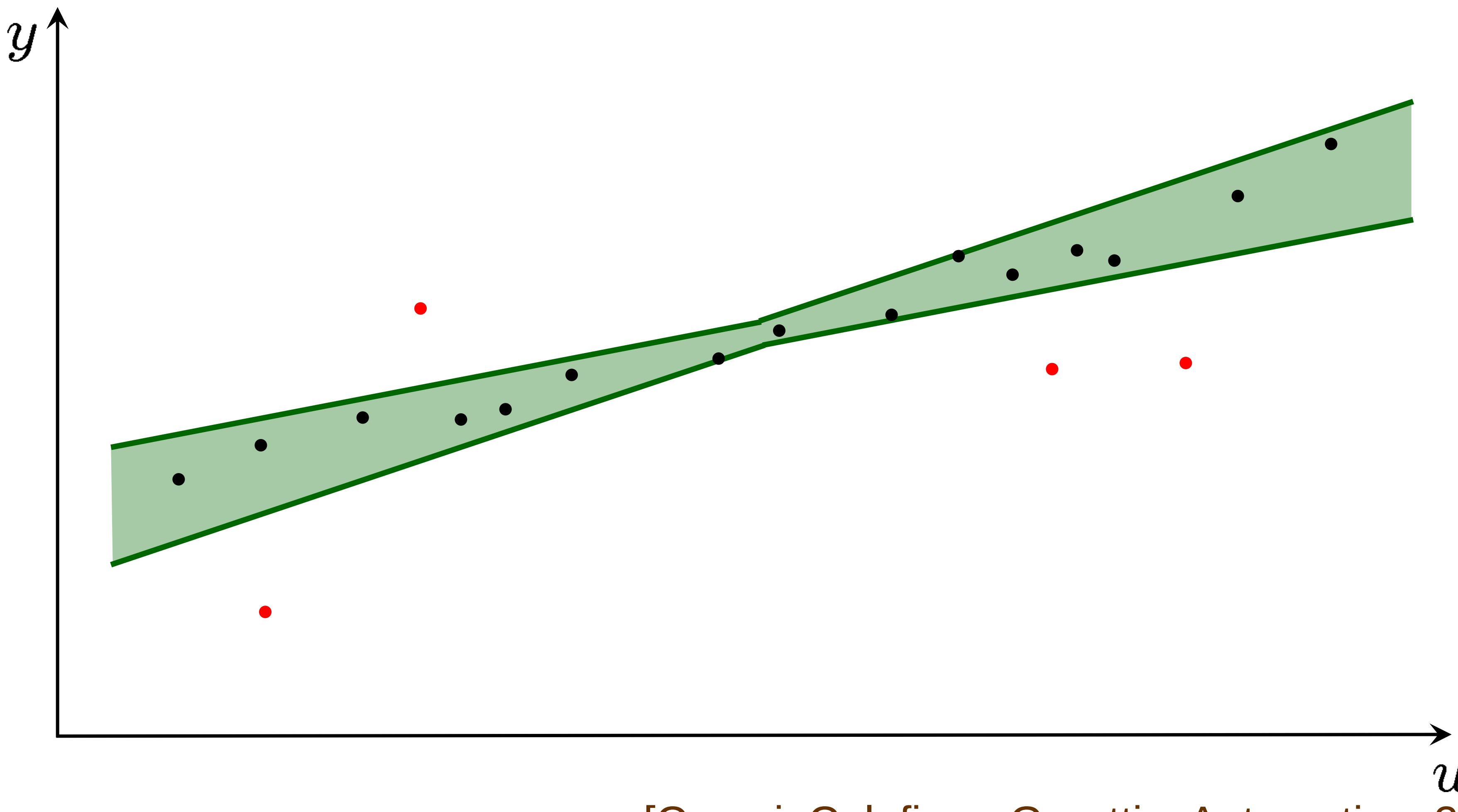

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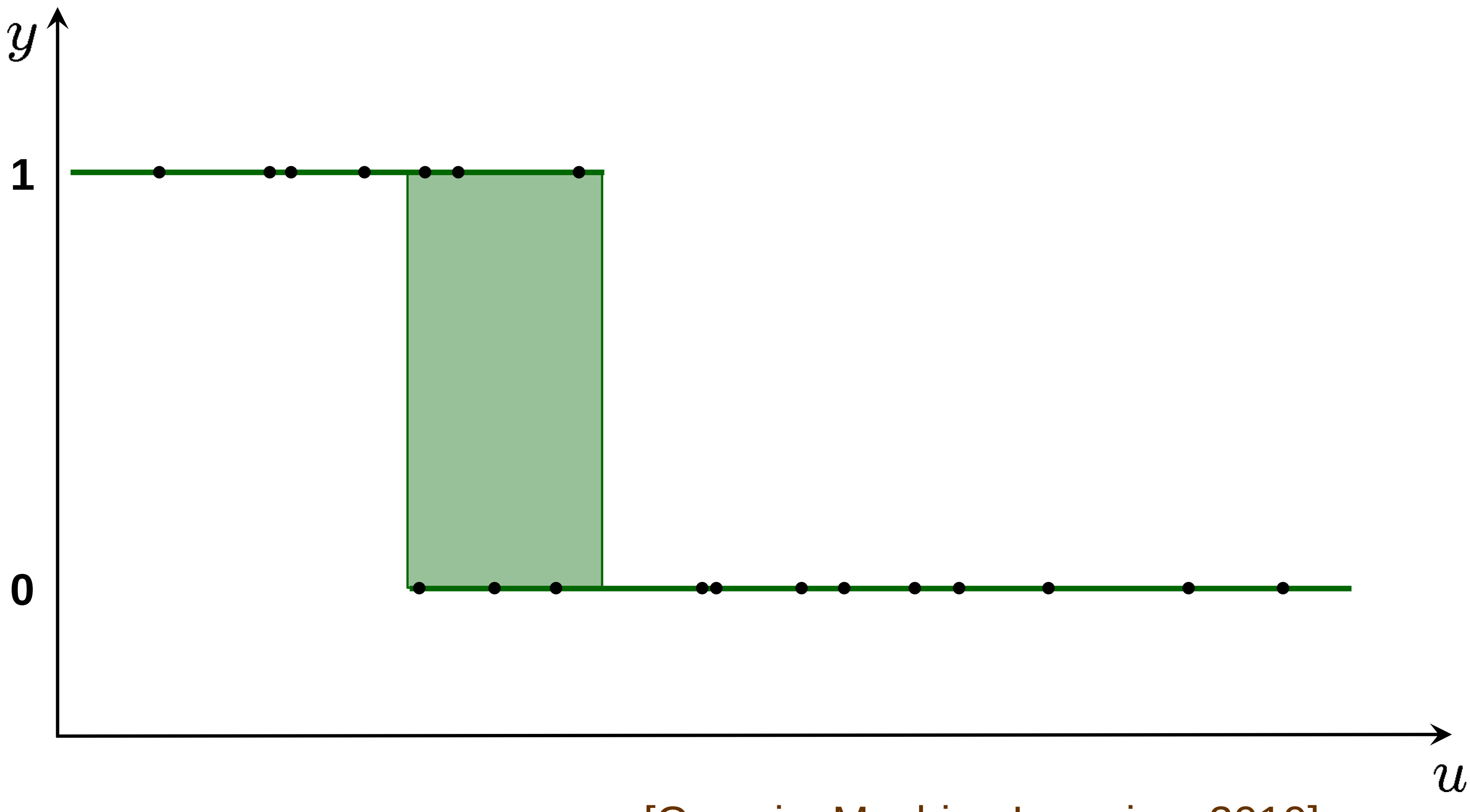
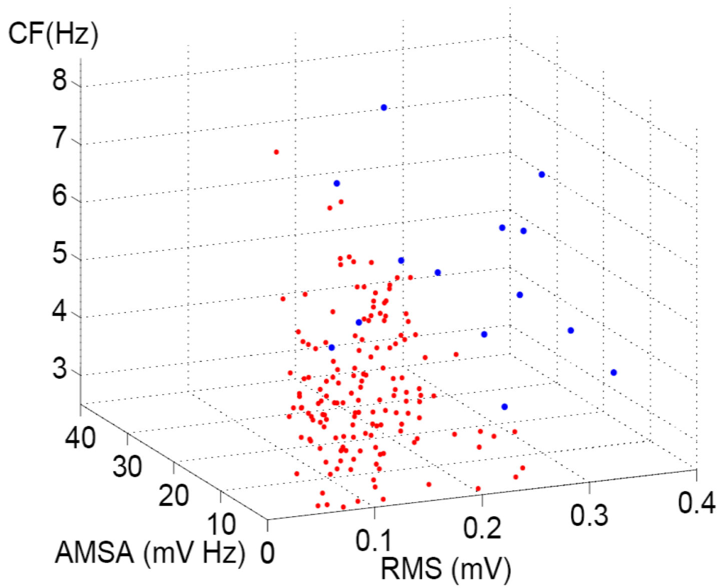
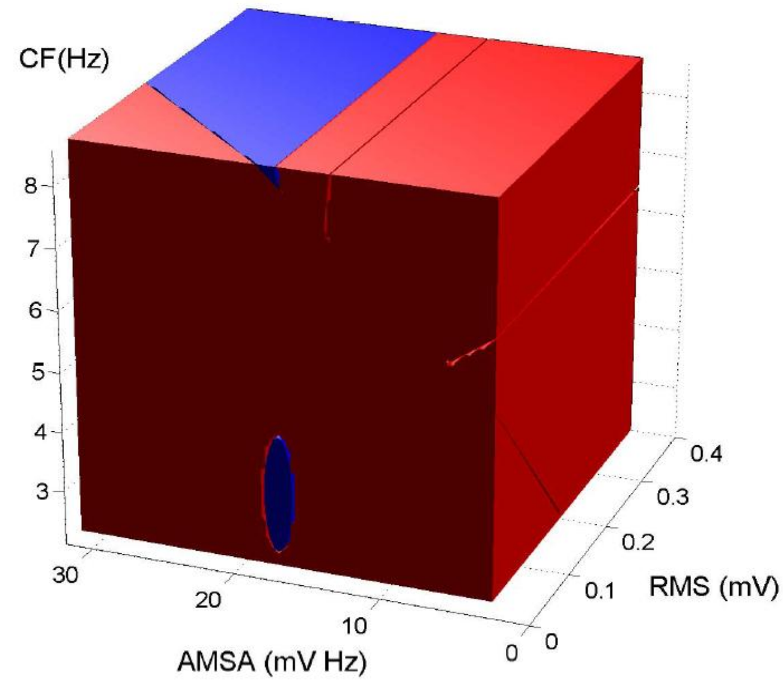


[Campi, Machine Learning, 2010]

example: defibrillation





[Campi, Machine Learning, 2010]

much remains to be done in prediction:

- more general constructions
- dynamical case

A journey through:

- parameter estimation
- prediction
- filtering

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beautiful opportunities for research!

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It's a wonderful world to explore, which offers incredible opportunities for research

Thank you!