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A new approach to controller design in presence of constraints

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Abstract— **In this paper, we present a new approach to control design in presence of constraints. This approach relies on the reformulation of the controller design problem as a semi-infinite convex optimization program, and on the solution of this program by the *scenario optimization technology*.**
The approach is illustrated through a simple example of disturbance rejection subject to input saturation constraints.

I. INTRODUCTION

In this paper, we propose a new approach to address robust control design in presence of constraints in a systematic and optimal way.

For ease of explanation, we illustrate this new approach through a simple example where, given a linear system affected by a disturbance belonging to some class, the goal is to design a feedback controller that attenuates the effect of the disturbance on the system output, while avoiding saturation of the control action due to actuator limitations.

The proposed control design method relies on the reformulation of the problem as a robust convex optimization program by adopting an appropriate parametrization of the controller. A robust convex optimization problem is expressed in mathematical terms as

$$\min_{\theta \in \Re^n} g(\theta) \text{ subject to:} \qquad (1)$$
$$f_\delta(\theta) \leq 0, \ \forall \delta \in \Delta,$$

where δ is the uncertain parameter, and $g(\theta)$ and $f_\delta(\theta)$ are convex functions in the n-dimensional optimization variable θ for every δ within the uncertainty set Δ. Convexity is appealing since 'convex' - as opposed to 'non-convex' - means 'solvable' in many cases, [1], [2]. In our context, the uncertain parameter δ represents a realization of the disturbance affecting the system, hence Δ contains an infinite number of instances. It is well known that *semi-infinite* optimization problems, that is problems with a finite number n of optimization variables and an infinite number of constraints, are difficult to solve and they have even proven NP-hard in some cases, [3], [4], [5], [6].

In [7], [8], an innovative technology called 'scenario approach' has been introduced to deal with semi-infinite convex programming at a very general level. The main thrust of this technology is that solvability can be obtained through random sampling of constraints provided that a probabilistic relaxation of the worst-case robust paradigm of (1) is accepted. Here, we propose to use the scenario technology for determining a solution to control design problems that would otherwise be hard to solve because of the presence of constraints and of uncertain signals/disturbances affecting the system. No extensions of the scenario approach itself are developed. Randomized algorithms for system analysis and control design have recently become a topic of great interest for the control community (see [9] for a comprehensive survey on the subject). Our contribution consists in the introduction of a novel randomized algorithm for robust control design in presence of constraints, which is based on the scenario approach.

In our control set-up where the uncertain parameter δ represents the disturbance realization, the implementation of the scenario optimization requires to randomly extract a certain number of disturbance realizations and to simulate the system behavior with the extracted realizations as input. This justifies the terminology we adopt to describe the proposed approach to control design as a 'simulation-based method'.

The problem of disturbance rejection has been addressed in the literature based on the dynamic programming approach [10], [11], [12], the l_1-optimal control theory [13], [14], the use of an upper bound on the l_1-norm (the star-norm) [15], and, more recently, through the invariant ellipsoids technique [16]. In all these approaches, the disturbance is only assumed to be bounded. Further possible knowledge on the disturbance signal (such as, for example, its correlation in time and main frequency components) is not exploited in the design process, which may lead to sub-optimal and conservative solutions for the problem at hand. Also, in the approaches based on dynamic programming and l_1-optimal control, the order of the controller cannot be fixed a-priori and the complexity of the resulting 'optimal' compensator may be high.

Other methodologies for solving quite general control design problems for linear systems affected by uncertain signals/disturbances and subject to constraints are present in the literature of receding horizon and model predictive control, [17], [18], [19], [20]. Differently from what we propose here, no structure is imposed to the feedback controller in these papers and design is carried out by directly optimizing over the control input samples in a time horizon of interest. The resulting feedback controller suffers from the problem to be difficult to implement, but it secures high performance under certain hypotheses. Moreover, applicability of standard methods in receding horizon model predictive

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control requires that uncertainty is quite structured (typically, the uncertain signals/disturbances are characterized through some polytopic or ellipsoidal bound on their instantaneous value), a limitation which is largely overcome by the approach proposed here.

The rest of the paper is organized as follows. In Section II, we precisely describe the control problem addressed and its reformulation as a semi-infinite convex optimization program. The application of the scenario technology to solve this optimization program is then explained in Section III, and a numerical example is provided in Section IV to illustrate the effectiveness of the resulting randomized method for control design. Some concluding remarks are drawn in Section V.

II. CONTROL PROBLEM FORMULATION

We consider a discrete time linear system with scalar input and scalar output, $u(t)$ and $y(t)$, governed by the following equation:

$$y(t) = G(z)u(t) + d(t), \tag{2}$$

where $G(z)$ is a stable transfer function and $d(t)$ is an additive disturbance.

Our objective is to determine a feedback control law

$$u(t) = C(z)y(t) \tag{3}$$

(see Figure 1) such that the disturbance $d(t)$ is optimally attenuated for every realization of $d(t)$ in some set of possible realizations $\mathcal{D}$, and such that the control input keeps within certain saturation limits. For example, $\mathcal{D}$ can be the set of step functions with specified maximum amplitude or the set of sinusoids with frequency in a certain range. A precise formalization of the optimization problem is next given.

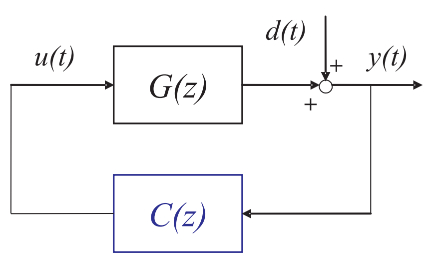

Fig. 1. The feedback disturbance compensation scheme.

Consider the finite-horizon 2-norm $\sum_{t=1}^{M} y(t)^2$ of the closed-loop system output. This norm quantifies the effect of the disturbance $d(t)$. For simplicity, we here consider (2) and (3) initially at rest, namely $G(z)u(t)$ represents an infinite backwards expansion $\sum_{j=1}^{\infty} g_j u(t-j)$ where $u(t-j) = 0$ for $t - j \leq 0$, and similarly for $C(z)y(t)$.

The goal is to minimize the worst-case disturbance effect

$$\max_{d(t)\in\mathcal{D}} \sum_{t=1}^{M} y(t)^2, \tag{4}$$

while maintaining the control input $u(t)$ within a saturation limit u_{bound}:

$$\max_{1\leq t\leq M} |u(t)| \leq u_{\text{bound}}, \; \forall d(t) \in \mathcal{D}. \tag{5}$$

Controller $C(z)$ is expressed in terms of an Internal Model Control (IMC) parametrization, [21]:

$$C(z) = \frac{Q(z)}{1 + Q(z)G(z)}, \tag{6}$$

where $G(z)$ is the system transfer function and $Q(z)$ is a free-to-choose transfer function (see Figure 2).

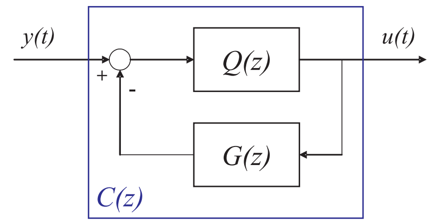

Fig. 2. The IMC parameterization of the controller.

Expression of $C(z)$ in (6) is totally generic, in that, given a $C(z)$, a $Q(z)$ can be always found generating that $C(z)$ through expression (6). The advantage of (6) is that the set of all controllers that closed-loop stabilize $G(z)$ is simply obtained from (6) by letting $Q(z)$ vary over the set of all stable transfer functions (see [21] for more details).

With (6) in place, the control input $u(t)$ and the controlled output $y(t)$ are given by:

$$u(t) = \frac{C(z)}{1 - C(z)G(z)} d(t) = Q(z)d(t) \tag{7}$$

$$y(t) = G(z)u(t) + d(t) = [G(z)Q(z) + 1]d(t). \tag{8}$$

The distinctive feature of these expressions is that $u(t)$ and $y(t)$ are affine in $Q(z)$. Consequently, if $Q(z)$ is selected from a family of stable transfer functions linearly parameterized in $\gamma := [\gamma_0\, \gamma_1\, \ldots \gamma_k]^T \in \Re^{k+1}$, i.e.

$$Q(z) = \gamma_0\beta_0(z) + \gamma_1\beta_1(z) + \gamma_2\beta_2(z) + \cdots + \gamma_k\beta_k(z), \tag{9}$$

where $\beta_i(z)$'s are pre-specified stable transfer functions, then the cost (4) and the constraints (5) are convex in γ.

A common choice for the $\beta_i(z)$'s functions is to set them equal to pure 'delays': $\beta_i(z) = z^{-i}$, leading to

$$Q(z) = \gamma_0 + \gamma_1 z^{-1} + \gamma_2 z^{-2} + \cdots + \gamma_k z^{-k}.$$

Another possibility is to let $\beta_i(z)$'s be Laguerre polynomials, [22], [23].

The control design problem can now be precisely formulated as follows:

$$\min_{\gamma,h\in\Re^{k+2}} h \quad \text{subject to:} \tag{10}$$

$$\sum_{t=1}^{M} y(t)^2 \leq h, \; \forall d(t) \in \mathcal{D}, \tag{11}$$

$$\max_{1\leq t\leq M} |u(t)| \leq u_{\text{bound}}, \; \forall d(t) \in \mathcal{D}. \tag{12}$$

Due to (11), h represents an upper bound to the output 2-norm $\sum_{t=1}^{M} y(t)^2$ for any realization of $d(t)$. Such an upper bound is minimized in (10) under the additional constraint (12) that $u(t)$ does not exceed the saturation limits.

We now rewrite problem (10)–(12) in a more explicit form.

By (7) and (8) and the parametrization of $Q(z)$ in (9), the input and the output of the controlled system can be expressed as

$$u(t) = \big(\gamma_o\beta_0(z) + \ldots + \gamma_k\beta_k(z)\big)d(t) \quad (13)$$

$$y(t) = G(z)\big(\gamma_o\beta_0(z) + \ldots + \gamma_k\beta_k(z)\big)d(t) + d(t). \quad (14)$$

Let us define the following vectors containing filtered versions of the disturbance $d(t)$:

$$\phi(t) := \begin{bmatrix} \beta_0(z)d(t) \\ \beta_1(z)d(t) \\ \vdots \\ \beta_k(z)d(t) \end{bmatrix} \text{ and } \psi(t) = \begin{bmatrix} G(z)\beta_0(z)d(t) \\ G(z)\beta_1(z)d(t) \\ \vdots \\ G(z)\beta_k(z)d(t) \end{bmatrix}. \quad (15)$$

Then, (13) and (14) can be re-written as

$$u(t) = \phi(t)^T\gamma$$
$$y(t) = \psi(t)^T\gamma + d(t),$$

and $\sum_{t=1}^{M} y(t)^2 = \gamma^T A\gamma + B\gamma + C$, where

$$A = \sum_{t=1}^{M} \psi(t)\psi(t)^T,\ B = 2\sum_{t=1}^{M} d(t)\psi(t)^T,\ C = \sum_{t=1}^{M} d(t)^2 \quad (16)$$

are matrices that depend on $d(t)$ only.

With all these positions, (10)–(12) rewrites as

$$\min_{\gamma,h\in\Re^{k+2}} h \text{ subject to:} \quad (17)$$
$$\gamma^T A\gamma + B\gamma + C \le h,\ \forall d(t) \in \mathcal{D}$$
$$-u_{\text{bound}} \le \phi(t)^T\gamma \le u_{\text{bound}},\ \forall t \in \{1,2,\ldots,M\},\ \forall d(t) \in \mathcal{D}.$$

Compared with the general form (1), the optimization variable θ is here (γ, h) and has size $n = k + 2$, and the uncertain parameter δ is the disturbance realization $d(t)$ taking value in the set $\Delta = \mathcal{D}$. Note that, given $d(t)$, quantities A, B, C, and $\phi(t)$ are fixed so that the first constraint in (17) is quadratic, while the others are linear.

Typically, the set $\mathcal{D}$ of disturbance realizations has infinite cardinality. Hence, problem (17) is a semi-infinite convex optimization problem.

III. Randomized solution through the scenario technology

As already pointed out in the introduction, semi-infinite convex optimization problems like (17) are difficult to solve. The idea of the scenario approach is that solvability can be recovered if some relaxation in the concept of solution is accepted. In the context of our control design problem, this means requiring that the constraints in (17) are satisfied for all disturbance realizations but a small fraction of them (*chance-constrained* approach).

The scenario approach goes as follows. Since we are unable to deal with the wealth of constraints in (17), we concentrate attention on just a few of them and extract at random N disturbance realizations $d(t)$ according to some probability distribution P introduced over $\mathcal{D}$. This probability distribution should reflect the likelihood with which the disturbance realizations occur or the relative importance that is attributed to different disturbance realizations. If no hint is available on which realization is more likely to occur and none of them is more critical than the others, then the uniform distribution can be adopted. A discussion on the use of the uniform distribution in randomized methods can be found in [24].

Only the extracted instances ('scenarios') are considered in the scenario optimization:

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extract N independent identically distributed realizations $d(t)_1,\ d(t)_2,\ \ldots,\ d(t)_N$ from $\mathcal{D}$ according to P. Then, solve the scenario convex program (SCP$_N$):

$$\min_{\gamma,h\in\Re^{k+2}} h \text{ subject to:} \quad (18)$$
$$\gamma^T A_i\gamma + B_i\gamma + C_i \le h,\ i = 1,\ldots,N,$$
$$-u_{\text{bound}} \le \phi(t)_i^T\gamma \le u_{\text{bound}},$$
$$\forall t \in \{1,2,\ldots,M\}, i = 1,\ldots,N,$$

where A_i, B_i, C_i, and $\phi(t)_i$ are as in (16) and (15) for $d(t) = d(t)_i$.

Letting (γ_N^*, h_N^*) be the solution to SCP$_N$, γ_N^* returns the designed controller parameter, whereas h_N^* quantifies the performance of the design compensator over the extracted disturbance realizations $d(t)_1,\ d(t)_2,\ \ldots,\ d(t)_N$.

The implementation of the scenario optimization requires that one picks N realizations of the disturbance and computes A_i, B_i, C_i, and $\phi(t)_i$ in correspondence of the extracted realizations. Since these quantities are artificially generated (that is they are not actual measurements coming from the system, but, instead, they are computer-generated), the proposed control design methodology can as well be seen as a *simulation-based approach*.

SCP$_N$ is a standard convex optimization problem with a finite number of constraints, and therefore easily solvable. On the other hand, it is spontaneous to ask: what kind of solution is one provided by SCP$_N$? Specifically, what can we claim regarding the behavior of the designed control system for all other disturbance realizations, those we have not taken into consideration while solving the control design problem? Answering this question is necessary to provide performance guarantees.

The above question is of the 'generalization' type in a learning-theoretic sense: we want to know how the solution (γ_N^*, h_N^*) generalizes in constraints satisfaction, from seen disturbance realizations to unseen ones. Certainly, any generalization result calls for some structure as no generalization is possible if no structure linking what has been seen to what has not been seen is present. The formidable fact in the context of convex optimization is that the solution of SCP_N always generalizes well, with no extra assumptions.

We have the following theorem (see Corollary 1 in [8]).

Theorem 1: Select a 'violation parameter' $\epsilon \in (0,1)$ and a 'confidence parameter' $\beta \in (0,1)$. Let $n = k+2$.
If

$$N = \left\lceil \frac{2}{\epsilon}\ln\frac{1}{\beta} + 2n + \frac{2n}{\epsilon}\ln\frac{2}{\epsilon} \right\rceil \tag{19}$$

($\lceil \cdot \rceil$ denotes the smaller integer greater than or equal to the argument), then, with probability no smaller than $1-\beta$, the solution (γ_N^*, h_N^*) to (18) satisfies all constraints of problem (17) with the exception of those corresponding to a set of disturbance realizations whose probability is at most ϵ. □

Let us read through the statement of this theorem in some detail. If we neglect the part associated with β, then, the result simply says that, by sampling a number of disturbance realizations as given by (19), the solution (γ_N^*, h_N^*) to (18) violates the constraints corresponding to other realizations with a probability that does not exceed a *user-chosen* level ϵ. This corresponds to say that – for other, unseen, $d(t)$'s – constraints (11) and (12) are violated with a probability at most ϵ. From (11) we therefore see that the found h_N^* provides an upper bound for the output 2-norm $\sum_{t=1}^{M} y(t)^2$ valid for any realizations of the disturbance with exclusion of at most an ϵ-probability set, while (12) guarantees that, with the same probability, the saturation limits are not exceeded.

As for the probability $1-\beta$, one should note that (γ_N^*, h_N^*) is a random quantity because it depends on the randomly extracted disturbance realizations. It may happen that the extracted realizations are not representative enough (one can even stumble on an extraction as bad as selecting N times the same realization!). In this case no generalization is certainly expected, and the portion of unseen realizations violated by (γ_N^*, h_N^*) is larger than ϵ. Parameter β controls the probability of extracting 'bad' realizations, and the final result that (γ_N^*, h_N^*) violates at most an ϵ-fraction of realizations holds with probability $1-\beta$.

In theory, β plays an important role and selecting $\beta = 0$ yields $N = \infty$. For any practical purpose, however, β has very marginal importance since it appears in (19) under the sign of logarithm: we can select β to be such a small number as 10^{-10} or even 10^{-20}, in practice zero, and still N does not grow significantly.

It is worth mentioning that improved bounds on the sample complexity N have been developed very recently in [25] and [26]. In particular, the bound derived in [26] is exact for the class of the so-called fully-supported problems.

IV. Numerical example

A simple example illustrates the controller design procedure.

With reference to (2), let

$$G(z) = \frac{0.2}{z-0.8},$$

and let the additive output disturbance be a piecewise constant signal that varies from time to time, at a low rate, of an amount bounded by some given constant. Specifically, let the set of admissible realizations $\mathcal{D}$ consists of piecewise constant signals changing at most once over any time interval of length 50, and taking value in $[-1,1]$.

As for the IMC parametrization $Q(z)$ in (9), we choose $k=1$ and $Q(z) = \gamma_0 + \gamma_1 z^{-1}$.

A control design problem (10)–(12) is considered with $M = 300$, and for two different values of the saturation limit u_{bound}: 10 and 1. Probability P is implicitly assigned by the recursive equation

$$d(t+1) = \big(1-\mu(t)\big)d(t) + \mu(t)v(t+1),$$

initialized with $d(1) = v(1)$, where $\mu(t)$ is a $\{0,1\}$-valued process ($\mu(t)=1$ at times where a jump occurs), and $v(t)$ is a sequence of i.i.d. random variables uniformly distributed in $[-1,1]$ ($v(t)$ is the new $d(t)$ value). $\mu(t)$ is generated according to

$$\mu(t) = \alpha(t)\prod_{k=1}^{50}\big(1-\mu(t-k)\big),$$

initialized with $\mu(0) = \mu(-1) = \cdots = \mu(-49) = 0$, where $\alpha(t)$ is a sequence of i.i.d. $\{0,1\}$-valued random variables taking value 1 with probability 0.01. An admissible realization of $d(t)$ in $\mathcal{D}$ is reported in Figure 3.

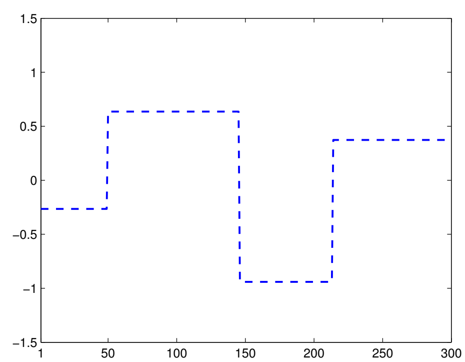

Fig. 3. A disturbance realization.

In the scenario approach we let $\epsilon = 5\cdot 10^{-2}$ and $\beta = 10^{-10}$. Correspondingly, N given by (19) is $N = 1370$.

From Theorem 1, with probability no smaller than $1 - 10^{-10}$, the obtained controller achieves the minimum of $\sum_{t=1}^{M} y(t)^2$ over all disturbance realizations, except a fraction of them of size smaller than or equal to 5%. At the same time, the control input $u(t)$ is guaranteed not to exceed the saturation limit u_{bound} except for the same fraction of disturbance realizations.

A. Simulation results

For $u_{\text{bound}} = 10$, we obtained $Q(z) = -4.993 + 4.024z^{-1}$ and, correspondingly, the transfer function $F(z) = 1 +$

$Q(z)G(z)$ between $d(t)$ and $y(t)$ (closed-loop sensitivity function) was

$$F(z) = 1 + (-4.993 + 4.024z^{-1})\frac{0.2}{z-0.8} \simeq 1 - z^{-1}.$$

The pole-zero plot of $F(z)$ is in Figure 4.

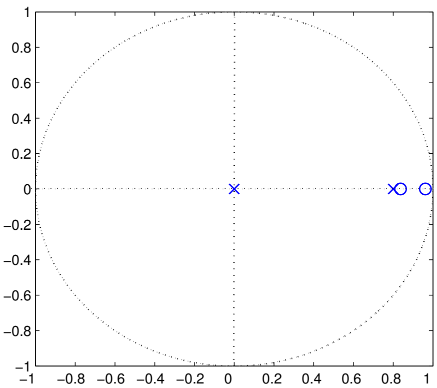

Fig. 4. Pole-zero plot of $F(z)$ when $u_{\text{bound}} = 10$. The poles are plotted as x's and the zeros are plotted as o's.

Since $y(t) = F(z)d(t) \simeq d(t) - d(t-1)$, then, when $d(t)$ has a step variation, $y(t)$ changes of the same amount and, when the disturbance gets constant, $y(t)$ is immediately brought back to zero and maintained equal to zero until the next step variation in $d(t)$ (see Figure 5). The obtained solution that $F(z)$ is approximately a FIR (Finite Impulse Response) of order 1 with zero DC-gain is not surprising considering that $d(t)$ varies at a low rate.

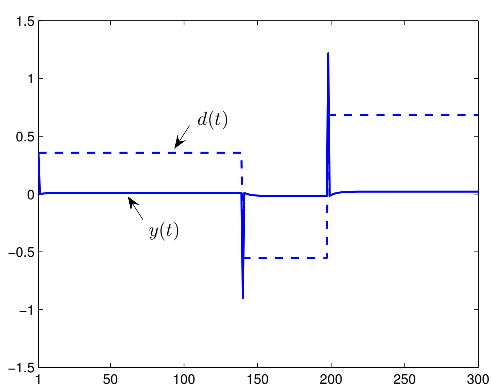

Fig. 5. Disturbance realization and corresponding output of the controlled system for $u_{\text{bound}} = 10$.

In the controller design just described, the limit $u_{\text{bound}} = 10$ played no role in that constraints $-u_{\text{bound}} \leq \phi(t)_i^T\gamma \leq u_{\text{bound}}$ in problem (18) were not active at the found solution. As u_{bound} is decreased, the saturation limits become more stringent and affect the solution.

For $u_{\text{bound}} = 1$, the following scenario solution was found $Q(z) = -0.991 + 0.011z^{-1}$, which corresponds to the sensitivity function:

$$F(z) = 1 + (-0.991 + 0.011z^{-1})\frac{0.2}{z-0.8} \simeq \frac{z-0.996}{z-0.8}.$$

The pole-zero plot of $F(z)$ is in Figure 6, while Figure 7 represents $y(t)$ obtained through this new controller for the same disturbance realization as in Figure 5. Note that the time required to bring $y(t)$ back to zero after a disturbance jump is now longer than 1 time unit, owing to saturation constraints on $u(t)$.

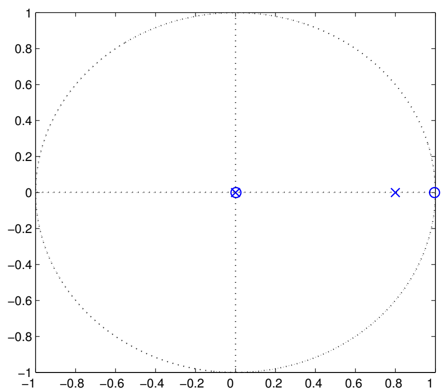

Fig. 6. Pole-zero plot of $F(z)$ when $u_{\text{bound}} = 1$. The poles are plotted as x's and the zeros are plotted as o's.

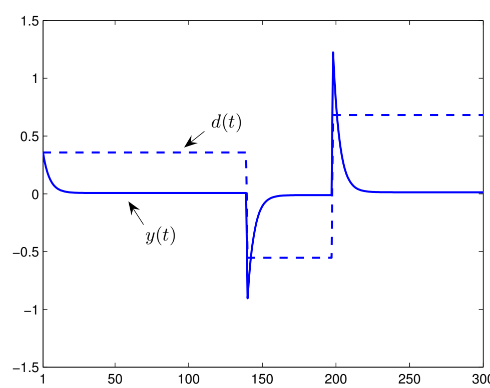

Fig. 7. Disturbance realization and corresponding output of the controlled system for $u_{\text{bound}} = 1$.

The optimal control cost value h_N^* is $h_N^* = 9.4564$ for $u_{\text{bound}} = 10$ and $h_N^* = 27.4912$ for $u_{\text{bound}} = 1$. As expected, the control cost increases as u_{bound} becomes more stringent.

The numerical example of this section is just one instance of application of the scenario approach to controller selection. The introduced methodology is of general applicability to diverse situations with constraints of different type, presence of reference signals, etc.

V. CONCLUSIONS

In this paper, we considered an optimal disturbance rejection problem with limitations on the control action and showed how it can be effectively addressed by means of the so-called scenario technology. This approach basically consists of the following main steps:

- reformulation of the problem as a robust (usually with infinite constraints) *convex* optimization problem;
- randomization over constraints and resolution (by means of standard numerical methods) of the so obtained *finite* optimization problem;
- evaluation of the constraint satisfaction level of the obtained solution through Theorem 1.

Extensions to tracking of some class of reference signals, and to control problems where the initial condition is uncertain or the output of the system is subject to some constraint are quite straightforward.

The applicability of the scenario methodology is not limited to optimal control problems with constraints and, indeed, this same methodology has been applied to a number of different endeavors in systems and control, [27], [28], [29], [30].

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