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Sensitivity shaping via Virtual Reference Feedback Tuning

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Abstract

The Virtual Reference Feedback Tuning (VRFT) is a data based method for the design of feedback controllers. In previous papers, the VRFT has been presented for the solution of the one degree of freedom model-reference control problem in which the objective is to shape the I/O transfer function of the control system. In this paper, the VRFT approach is extended so that it can be used for the shaping of the sensitivity function.

1 Introduction

We consider the problem of designing a feedback controller for a linear plant, whose transfer function $P(z)$ is *unknown*. The design is based on a set of I/O data (see also [6, 9, 4]).
The Virtual Reference Feedback Tuning (VRFT) method [3, 7] gives a solution to the above problem without resorting to the identification of a model of the plant. The idea on which VRFT is based was originally proposed in [5]. In [2, 3, 7] the VRFT was developed as a complete and ready to use method for the design of the controller with the objective of shaping the complementary sensitivity of the designed control system. The aim of this paper is to show how the same approach can be used for the design of a feedback controller in order to impose the output sensitivity transfer function.

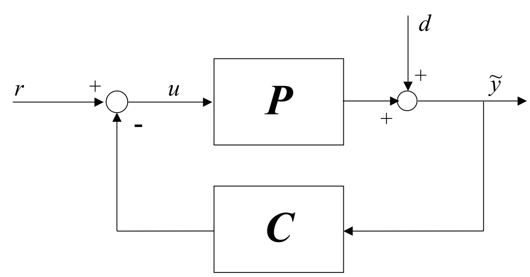

Figure 1: The feedback control system.

Problem formulation
It is assumed that the plant is a *deterministic* and *linear* SISO discrete-time dynamical system described by the rational transfer function $P(z)$. Such a transfer function is *unknown* and we cannot make use of the knowledge of $P(z)$ in the controller design. Instead we will resort to a set of data collected during an experiment on the plant. We assume that an additive noise signal $d(t)$ affects the output of the plant.
The following one degree of freedom control system - see fig.(1) - is considered:

$$\begin{cases} \tilde{y}(t) = P(z)u(t) + d(t) \\ u(t) = -C(z;\theta)\tilde{y}(t) + r(t) \end{cases}$$

in which $C(z;\theta)$ is a linear controller belonging to a given family $\{C(z;\theta)\}$ of controllers parameterized by θ, $d(t)$ is a disturbance signal and

$r(t)$ is an external reference signal (normally left at zero). The control problem addressed herein is the shaping of the sensitivity function with a model-reference criterion. Namely, given a reference sensitivity model $S(z)$ select the parameter vector $\bar{\theta}$ so as to minimize the following model-reference criterion:

$$J_{MR}(\theta) = \left\| \left(\frac{1}{1 + P(z)C(z;\theta)} - S(z) \right) W(z) \right\|_2^2 \tag{1}$$

where $W(z)$ is a user-chosen weighting function.

The attention is restricted to linearly parameterized controllers of the form $C(z;\theta) = \beta(z)^T\theta$ in which: $\beta(z)$ is a vector of discrete-time transfer functions and $\theta \in \mathbf{R}^n$ is the n-dimensional vector of parameters.

Outline of the paper

In Section 2 the Virtual Reference idea for the shaping of the sensitivity is introduced. Based on this idea a design algorithm is formulated. An analysis of the achievable performance is given in Section 3. For the sake of exposition clarity, in Section 2 and 3 it is assumed that the collected data are not corrupted by noise (i.e. $d(t) = 0$). The use on noisy data is described in Section 4. A simulation example, in Section 5, ends the paper.

2 The Virtual Reference framework for the shaping of the sensitivity

The Virtual Reference idea

Let us assume that a set of data $\{u(t), y(t)\}_{t=1,..,N}$ has been collected from a noise-free experiment on the plant. Given the measured $y(t)$, construct a signal $\bar{d}(t)$ such that $y(t) + \bar{d}(t) = S(z)\bar{d}(t)$. Next, calculate the signal $\bar{y}(t) = y(t) + \bar{d}(t)$ and notice that: if the disturbance signal is $\bar{d}(t)$ and $r(t) = 0$ then $\bar{y}(t) = S(z)\bar{d}(t)$ is the desired output of the closed-loop system and, moreover, $y(t)$ is the desired output of the plant (see fig.(2)).

Even though plant $P(z)$ is not known, we know that when $P(z)$ is fed by $u(t)$ (the actually measured input signal), it generates $y(t)$ as an output. Therefore, we can state that a good feedback controller $C(z)$, at least in the case in which the disturbance is $\bar{d}(t)$ and $r(t) = 0$, is one that generates $u(t)$ when fed by $\bar{y}(t)$. The idea is to search for such a controller. Since $u(t)$ and $\bar{y}(t)$ are known signals, the controller can be identified from these signals.

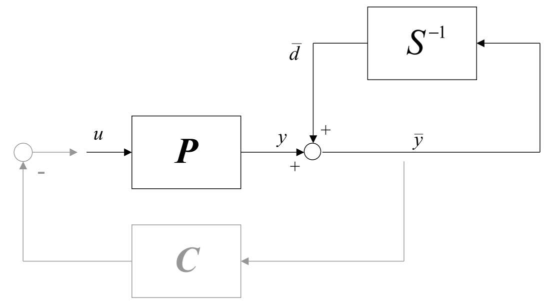

Figure 2: The construction of $\bar{d}(t)$ and $\bar{y}(t)$.

The design algorithm

We implement the above idea in the following algorithm. In the algorithm we include also a possible filtering action whose choice will be discussed later.

Algorithm 1

Given the reference sensitivity model $S(z)$, the family of available controllers $\{C(z;\theta)\}$ and the set of data $\{u(t), y(t)\}_{t=1,..,N}$, do the following:

1. Calculate:
   - a signal $\bar{d}(t)$ such that $y(t) + \bar{d}(t) = S(z)\bar{d}(t)$,
   - and $\bar{y}(t) = y(t) + \bar{d}(t)$.
2. Filter the signals $\bar{y}(t)$ and $u(t)$ with a suitable filter $L(z)$:

   $$\bar{y}_L(t) = L(z)\bar{y}(t), \qquad u_L(z) = L(z)u(t);$$

3. Select the controller parameter vector, say $\hat{\theta}_N$, that minimizes the following performance index $J_{VR}^N(\theta)$:

   $$J_{VR}^N(\theta) = \frac{1}{N}\sum_{t=1}^{N}(u_L(t) + C(z;\theta)\bar{y}_L(t))^2. \tag{2}$$

Note that when the controller has the form $C(z;\theta) = \beta^T(z)\theta$, the performance index (2) is quadratic and the parameter vector $\hat{\theta}_N$ is easily obtained by solving the normal equations:

$$\hat{\theta}_N = \left[\frac{1}{N}\sum_{t=1}^{N}\varphi_L(t)\varphi_L(t)^T\right]^{-1}\left[-\frac{1}{N}\sum_{t=1}^{N}\varphi_L(t)u_L(t)\right]. \tag{3}$$

3 Analysis of the design criterion

In the following, the performance index of the model-reference control problem (1) and the design criterion of the Virtual Reference approach (2) will be compared. Even if they look different, it will be shown that their minimum arguments can be made close to each other by a suitable selection of the filter $L(z)$. In this way, the Virtual Reference approach can be used to solve, at least approximately, the model-reference control problem stated in the introduction.

In order to give such a comparison, we introduce the ideal controller $C_0(z)$ which exactly solves the model matching problem. Namely, $C_0(z)$ is given by:

$$C_0(z) = \frac{1}{P(z)} \frac{1 - S(z)}{S(z)} \quad (4)$$

Notice that $C_0(z)$ is clearly an *ideal* controller. In general it does not belong to the available family of parameterized controllers $\{C(z;\theta)\}$. Even more so it should result to be not proper, that is, strictly speaking, it should not be a transfer function. Therefore, $C_0(z)$ is not expected to belong to the available family of parameterized controllers $\{C(z;\theta)\}$ in practice. The ideal controller $C_0(z)$ will be used in the following only as an analysis tool.

To start with, note that, using the definition of 2-norm of a discrete-time linear transfer function and the definition of the ideal controller $C_0(z)$,the model-reference criterion $J_{MR}(\theta)$ can be written as:

$$J_{MR}(\theta) = \frac{1}{2\pi} \int_{-\pi}^{\pi} |P|^2 \frac{|C_0 - C(\theta)|^2}{|1 + PC(\theta)|^2} |S|^2 |W|^2 d\omega. \quad (5)$$

(throughout, we will drop the argument $e^{j\omega}$ of transfer function). The minimum point of $J_{MR}(\theta)$ is indicated as $\bar{\theta}$.

Consider now the criterion $J_{VR}^N(\theta)$, under the hypothesis of ergodicity of the involved signals, using the Parseval theorem (see [8]) and the definition of $C_0(z)$ the following asymptotic (as $N \to \infty$) frequency domain representation of $J_{VR}^N(\theta_r, \theta_y)$ can be written:

$$J_{VR}(\theta) = \frac{1}{2\pi} \int_{-\pi}^{\pi} |P|^2 |C(\theta) - C_0|^2 \frac{|S|^2 |L|^2}{|S - 1|^2} \Phi_u d\omega \quad (6)$$

where Φ_u is the power spectral density of $u(t)$. The minimum of $J_{VR}(\theta)$ is indicated as $\hat{\theta}$ (the parameter vector $\hat{\theta}_N$ will converge to $\hat{\theta}$ as $N \to \infty$). For analysis purposes, $J_{VR}(\theta)$ will be used extensively in place of $J_{VR}^N(\theta)$.

Notice, by comparing (5) and (6), that, if $C_0(z) \in \{C(z;\theta)\}$, then the minimum of $J_{VR}(\theta)$ corresponds to the ideal controller and coincides with the minimum of $J_{MR}(\theta)$, whatever the plant, the filter and the reference model are. Therefore, in the ideal case in which the ideal controller belongs to the class of available controllers, the controller estimated through the Virtual Reference approach coincides with the ideal one. On the other hand, it is apparent that $J_{VR}(\theta)$ and $J_{MR}(\theta)$ have different minimum points when the class of available controllers has restricted complexity (i.e. $C_0(z) \notin \{C(z;\theta)\}$).

In the following we present a result showing that in general the minimum arguments of $J_{VR}(\theta)$ and $J_{MR}(\theta)$ can be made close to each other by a suitable selection of the filter $L(z)$.

The choice of the filters

The following choice of the filter is here proposed: select $L(z)$ such that

$$|L|^2 = |S - 1|^2 |S|^2 |W|^2 \frac{1}{\Phi_u} = \frac{|S - 1|^2 |W|^2}{|1 + PC_0|^2} \frac{1}{\Phi_u}. \quad (7)$$

Notice that all quantities in the right-hand-side of equations (7) are known and therefore $L(z)$ can be actually computed. The only $\Phi_u(\omega)$ may be considered known only in certain situations, when the input signal has been selected by the designer; otherwise $\Phi_u(\omega)$ can be estimated using many different techniques, among which a high-order AR or ARX model, or a high-order state-space model - see [8].

In the following Proposition (1), we show that choice (7) is optimal in a certain sense. Before stating this result, some notations must be preliminary settled.

Set

$$\begin{aligned}
\Delta C(z) &= C_0(z) - \beta^T(z)\bar{\theta} \\
\beta^+(z) &= [\beta_1(z) \quad \beta_2(z) \quad \ldots \quad \beta_n(z) \quad \Delta C(z)]^T \\
\theta^+ &= [\vartheta_1 \quad \vartheta_2 \quad \ldots \quad \vartheta_n \quad \vartheta_{n+1}]^T;
\end{aligned}$$

where $\bar{\theta}$ is the parameter vector which minimizes $J_{MR}(\theta)$; then define an extended family of controllers as follows:

$$C^+(z;\theta^+) = \beta^+(z)^T \theta^+.$$

Clearly, $C_0(z) \in \{C^+(z;\theta^+)\}$ with $\bar{\theta}^+ = [\bar{\theta}^T \quad 1]^T$. Finally, consider the extended performance index

$$J^+_{MR}(\theta^+) = \left\| \left(\frac{1}{1+P(z)C^+(z;\theta^+)} - S(z) \right) W(z) \right\|_2^2.$$

and the second order Taylor expansion around its global minimizer $\bar{\theta}^+$ which is denoted by $\tilde{J}^+_{MR}(\theta^+)$, namely:

$$J^+_{MR}(\theta^+) = \tilde{J}^+_{MR}(\theta^+) + o(\|\theta^+ - \bar{\theta}^+\|_2^2).$$

Using the above definitions, Proposition (1) can now be stated.

Proposition 1

The parameter vector $\bar{\theta}$ which minimizes the performance index $J_{MR}(\theta)$, and the parameter vector $\hat{\theta}$ which minimizes the performance index $J_{VR}(\theta)$ when $L(z)$ is selected as in (7) are such that:

$$\bar{\theta} = \arg\min_\theta J^+_{MR}([\theta^T \quad 0]^T);$$

$$\hat{\theta} = \arg\min_\theta \tilde{J}^+_{MR}([\theta^T \quad 0]^T).$$

proof: *see [2].* □

Discussion

The above result is interesting since it provides a formal relationship between the parameter vector $\hat{\theta}$ obtained using the Virtual Reference approach (in the special case when $L(z)$ is selected according to (7)), and the "optimal" parameter vector $\bar{\theta}$, which minimizes the original performance index $J_{MR}(\theta)$. Based on this result, we conclude that if the transfer function $\Delta C(z)$ plays a marginal role in determining $C_0(z)$, namely the family of controllers $\{C(z;\theta)\}$ is only slightly underparameterized given a certain reference sensitivity model, then $C(z;\hat{\theta})$ is a good approximation to $C(z;\bar{\theta})$ since $J^+_{MR}(\theta^+)$ is well approximated in a neighborhood of its minimum by its second order expansion $\tilde{J}^+_{MR}(\theta^+)$.

4 The use of noisy data

It has been shown that the controller can be estimated, on the basis of a set of noise-free data, by solving an ad-hoc identification problem. Now let us assume that a set $\{u(t), \tilde{y}(t)\}_{t=1,..,N}$ of data has been collected from an open-loop noisy experiment. We make the standard assumption that the signal $u(t)$ and the noise signal $d(t)$ are ergodic and uncorrelated signals.

If one applies the Algorithm 1 to the set $\{u(t), \tilde{y}(t)\}_{t=1,..,N}$ of noisy data then he obtains a perturbed version, say $\tilde{\theta}_N$, of the right parameters vector $\hat{\theta}_N$. The reason is that the regression vector in (3), now constructed from $\tilde{y}(t)$, namely:

$$\tilde{\varphi}_L(t) = \beta(z)L(z)S(z)(S(z)-1)^{-1}\tilde{y}(t),$$

is corrupted by noise. Due to the presence of noise, in the limit we have:

$$\lim_{N\to\infty} \tilde{\theta}_N \neq \hat{\theta} = \lim_{N\to\infty} \hat{\theta}_N.$$

In the following, we propose the use of an Instrumental Variable method to counteract the effect of noise ([8]). In this way, the results given in previous sections can be saved for the case in which the data are corrupted by noise.

Instrumental Variables

The idea behind the Instrumental Variable method is to use a generic regression vector $\zeta(t)$, uncorrelated with the noise but correlated with $\tilde{\varphi}_L(t)$, and estimate the parameter vectors as follows:

$$\hat{\theta}^{IV}_N = \left[\frac{1}{N} \sum_{t=1}^N \zeta(t)\tilde{\varphi}_L(t)^T \right]^{-1} \left[-\frac{1}{N} \sum_{t=1}^N \zeta(t) u_L(t) \right]. \quad (8)$$

The elements of $\zeta(t)$ are called instrumental variables. If the instrumental variables are uncorrelated with the noise then the limit estimate $\hat{\theta}^{IV} = \lim_{N\to\infty} \hat{\theta}^{IV}_N$ does not depends on the noise. We can say that the noise is "correlated out" by the instrumental variables. However, the quality of the estimate depends on the choice of the instrumental variables. The more $\zeta(t)$ is correlated with $\varphi_L(t)$, the more $\hat{\theta}^{IV}$ is close to $\hat{\theta}$. Some appropriate choices are discussed below.

Choice of the instrumental variables

We propose two different choices for the instrumental variables. The first guarantees that asymptotically $\hat{\theta}^{IV} = \hat{\theta}$. However an additional experiment on the plant is required. The second does not guarantee that asymptotically $\hat{\theta}^{IV} = \hat{\theta}$ but the residual error is expected to be small. This choice does not require an additional experiment on the plant but calls for additional computational effort since it requires a model fitting step. The proposed choices are as follows:

- **Repeated experiment**
  Given the data $\{u(t), \tilde{y}(t)\}_{t=1,..,N}$, repeat the experiment on the plant using the same input $\{u(t)\}_{t=1,..,N}$ and collect the corresponding output sequence $\{\tilde{y}(t)'\}_{t=1,..,N}$. Then construct the instrumental variables

as:

$$\zeta(t) = \beta(z)L(z)S(z)(S(z)-1)^{-1}\tilde{y}'(t) \quad (9)$$

Notice that $\{\tilde{y}(t)'\}_{t=1,..,N}$ will be different from $\{\tilde{y}(t)\}_{t=1,..,N}$ since the two sequences are affected by two different realizations of the noise coming from two distinct experiments. It is reasonable to assume that the noise signals in the two experiments are uncorrelated.

- **Identification of the plant**
  Identify a model $\hat{P}(z)$ of the plant from the set of data $\{u(t), \tilde{y}(t)\}_{t=1,..,N}$ and generate the simulated output $\hat{y}(t) = \hat{P}(z)u(t)$. Then construct the instrumental variables as:

$$\zeta(t) = \beta(z)L(z)S(z)(S(z)-1)^{-1}\hat{y}(t) \quad (10)$$

  The identification of the plant is a standard open-loop identification problem. The model $\hat{P}(z)$ can be estimated using different techniques - see [8]. Notice that, since the model $\hat{P}(z)$ is used only as a simulator of the plant, there are no strict limitations to its order (because it is not directly involved in the design of the controller). As a consequence, high-order models can be used to achieve high accuracy.

The choices of the instrumental variables proposed in (9) and (10) are analyzed in the following proposition.

Proposition 2
Assume that: (i) the data $\{u(t), \tilde{y}(t)\}_{t=1,..,N}$ and the instrumental variables $\{\zeta(t)\}_{t=1,..,N}$ are realizations of ergodic stochastic processes; (ii) $E[\zeta(t)\varphi_L(t)] > 0$; (iii) $E[\zeta(t)d(t)] = 0$; then

- if $\zeta(t)$ is chosen as in (9) then $\hat{\theta}^{IV}$ satisfies

$$\hat{\theta}^{IV} = \hat{\theta} \quad \text{with probability 1.}$$

- if $\zeta(t)$ is chosen as in (10) then $\hat{\theta}^{IV}$ minimizes (with probability 1) the following quadratic criterion:

$$\hat{J}_{VR}(\theta) = \frac{1}{2\pi}\int_{-\pi}^{\pi} P\overline{\hat{P}}|C(\theta)-C_0|^2 \frac{|S|^2|L|^2}{|S-1|^2}\Phi_u d\omega$$

Proof: *see [7]* □

Closed-loop noisy data
The VRFT method can be successfully applied to data collected in closed-loop as well. An extended discussion on the use of closed-loop data goes beyond the scope of this paper. Here, it suffices to say that the above instrumental variable procedure can still be applied by replacing the repeated open-loop experiment with a repeated closed-loop experiment with the same reference signal (or the identification of $\hat{P}(z)$ with the identification of the complementary sensitivity of the closed-loop system). The interested reader is referred to [7].

5 A Numerical Example

The following continuous-time plant is considered (from [1]):

$$P(s) = \frac{\mu}{1 + (2\zeta/\omega_n)s + (1/\omega_n^2)s^2}$$

where: $\mu = 0.63$, $\omega_n = 304$, $\zeta = 0.18$. The plant is sampled at $T_s = \pi/10^4$ s. The magnitude Bode plot of $P(s)$ is shown in Fig.(3). The output of the plant if affected by an additive disturbance signal $d(t)$ having the following form:

$$d(t) = \sin(\omega_d \cdot t) + \nu(t)$$

where the frequency of the sinusoidal component $\omega_d = 2\pi \cdot 38$ rad/s in known and $\nu(t)$ is white noise with variance $\sigma_\nu^2 = 10^{-4}$. The design objective is to design a controller for disturbance rejection. The following sensitivity reference model expresses the desired performance:

$$S(z) = \left(1 - \frac{(1-\alpha)z^{-1}}{1-\alpha z^{-1}}\right) \cdot H(z)$$

where $\alpha = e^{-\bar{\omega}T_s}$, $\bar{\omega} = 2\pi \cdot 100$ rad/s and $H(z)$ is a notch filter centered on $e^{-j\omega_d T_s}$ (see [1] and the works cited therein):

$$H(z) = c \cdot \frac{1 - 2\cos(\omega_d T_s)z^{-1} + z^{-2}}{1 - 2\rho\cos(\omega_d T_s)z^{-1} + \rho^2 z^{-2}}$$

where $\rho = 0.98$ and $\bar{c}$ is such that $\bar{C}(z)$ has unitary gain. The Magnitude bode plot of the reference model is shown in Fig.(3). The weighting factor has been chosen as: $W_(z) = 1$. The following class of controllers has been adopted:

$$C(z;\theta) = \frac{\vartheta_0 + \vartheta_1 z^{-1} + \vartheta_2 z^{-2} + \vartheta_3 z^{-3}}{1 - z^{-1}} \cdot \bar{C}(z)$$

$$\bar{C}(z) = \bar{c} \cdot \frac{1 - 2\rho\cos(\omega_d T_s)z^{-1} + \rho^2 z^{-2}}{1 - 2\cos(\omega_d T_s)z^{-1} + z^{-2}}$$

where $\bar{c}$ is such that $\bar{C}(z)$ has unitary gain. In the controller a fixed integral action is included and, moreover poles, corresponding to the zeros of the notch filter $H(z)$, are included in order to meet the specification on the disturbance rejection signal. A set $\{u(t), \tilde{y}(t)\}_{t=1,..,N}$ of open-loop noisy data has been collected by feeding the plant with a white noise signal ($\Phi_u(\omega) = 1$) for 1 s. The parameter vector $\hat{\theta}_N^{IV}$ have been estimated from the data through the Virtual Reference method. The optimal filter (7) has been chosen. The data have pre-filtered through $H(z)$ in order to eliminate the sinusoidal component of the disturbance. As for the stochastic components, a set of data, obtained by repeating the experiment on the plant with the same input, has been used in order to construct the instrumental variables (see Section 4). The estimated parameters vectors was: $\hat{\theta}_N^{IV} = [62.015 \; -142.614 \; 101.534 \; -20.59]$ The achieved control system is illustrated below. In Fig.(4.a) the magnitude bode plot of the sensitivity transfer function of the achieved control system is compared with the one of the reference model. In Fig.(4.b) the response of the control system to the disturbance signal is shown.

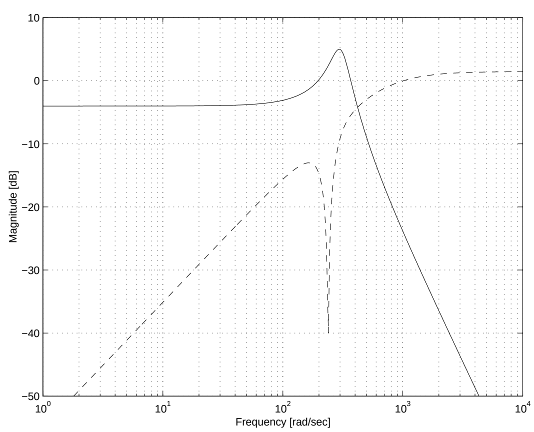

Figure 3: Magnitude Bode plots: $|P(j\omega)|$ (continuous line), and $|S(e^{j\omega T_s})|$ (dotted line).

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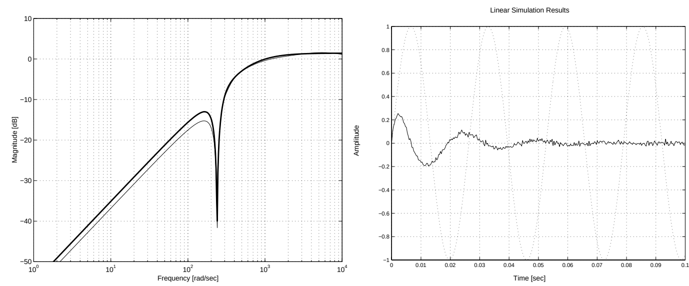

(a) (b)

Figure 4: **(a)** Magnitude bode plots: the achieved sensitivity transfer function (thin line) and the sensitivity reference-model (bold line); **(b)** Response of the designed sensitivity function to the disturbance signal: disturbance signal (dotted line) and output signal (continuous line).

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