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Self-Tuning Control based on Penalized Least Squares Identification Techniques

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Abstract

In self-tuning control problems, the parameter estimate can exhibit undesirable behaviors due to the possible lack of information in some directions of the parameter space. In this contribution, a penalized technique able to enforce certain desired properties to the estimates is introduced. Its usefulness in an adaptive control context is also discussed.

1 The identification algorithm

Consider the discrete time SISO system governed by the equation

$$A(\theta^o;q^{-1})\,y_t = B(\theta^o;q^{-1})\,u_{t-d} + n_t, \qquad d \geq 1 \qquad (1)$$

where

$$A(\theta^o;q^{-1}) = 1 - \sum_{i=1}^{n} a_i^o q^{-i}$$

and

$$B(\theta^o;q^{-1}) = \sum_{i=0}^{m} b_i^o q^{-i}$$

are polynomials in the unit-delay operator q^{-1} and $\theta^o = [a_1^o\ a_2^o \dots a_n^o\ b_0^o\ b_1^o \dots b_m^o]^T$ is the system parameter vector.

Signal n_t is a stochastic disturbance precisely described in the following

Assumption 1 $\{n_t\}$ is i.i.d. and normally distributed with $E[n_t] = 0$ and $E[n_t^2] = \sigma^2 > 0$. □

Moreover, we assume that system (1) is controllable according to

Assumption 2 $q^s A(\theta^o;q^{-1})$ and $q^{s-d}B(\theta^o;q^{-1})$ are coprime, where $s = \max(n, m+d)$. □

By letting $\varphi_t = [y_t \dots y_{t-(n-1)}\ u_{t-d+1} \dots u_{t-(m+d-1)}]^T$ be the observation vector, system (1) can be given the form $y_t = \varphi_{t-1}^T\theta^o + n_t$ and the standard cost for the least squares (LS) algorithm can be written as

$$V_t(\theta) = \sum_{s=1}^{t}(y_s - \varphi_{s-1}^T\theta)^2,$$

the minimizer of which will be denoted by $\hat{\theta}_t^{LS}$.

The main properties of the least squares estimate relevant for adaptive control systems analysis can be put sharply into focus thanks to the notion of *excitation subspace*.

Definition 1 (*excitation subspace*, [1])
The subspace $\bar{\mathcal{E}} = \{x \in R^{n+m+1} / \exists L < \infty : x^T \sum_{s=1}^{N} \varphi_{s-1}\varphi_{s-1}^T\, x < L,\ \forall N\}$ is termed *unexcitation subspace*. Its orthogonal complement $\mathcal{E} = \bar{\mathcal{E}}^{\perp}$ is called *excitation subspace*. □

Theorem 1 (*properties of the LS estimate*, [2]-[4])
The estimate $\hat{\theta}_t^{LS}$ is asymptotically convergent almost surely. Moreover, denoting by $\hat{\theta}_{U,t}^{LS}$ and $\hat{\theta}_{E,t}^{LS}$ the components of $\hat{\theta}_t^{LS}$ along the unexcitation and excitation subspaces respectively, we have:

i) $\lim_{t\to\infty} \hat{\theta}_{U,t}^{LS} = \hat{\theta}_{U,\infty}^{LS}$ ($\neq \theta_U^o$ in general) almost surely.

ii) $\lim_{t\to\infty} \hat{\theta}_{E,t}^{LS} = \hat{\theta}_{E,\infty}^{LS} = \theta_E^o$ almost surely. □

In particular, property *ii)* in Theorem 1 says that the component of the true parameterization θ^o along the excitation directions is asymptotically consistently estimated (*partial consistency*). Clearly, this is exactly what is needed for control purposes, since, in view of the very definition of excitation subspace, θ_E^o is the only component of θ^o excited by data in the long run.

On the other hand, it is often the case that one wishes to enforce in the estimate some additional constraints so as to satisfy specific requirements besides the partial consistency property. A pair of significant instances are given below.

- In adaptive control, a suitable control law can often be found only under the constraint that the estimated model is controllable (*i.e.* it does not present pole-zero cancellations). Therefore, it is desirable to avoid those parameterizations which correspond to uncontrollable models.
- In practice, the parameters to be estimated have often a physical interpretation. In such a case, one may desire to incorporate in the estimation algo-

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rithm possible a-priori knowledge on their real value.

In this paper, we propose a *penalized identification technique* with a twofold objective:

- *enforcing specific constraints* on the *feasible range* for the parameter estimates;
- *preserving the partial consistency property* of the standard least squares algorithm.

With this objective in mind, we introduce the following algorithm. Consider a penalty function $P(\theta)$ satisfying the conditions:

i) $P(\theta) \geq 0$, $\forall \theta \in R^{n+m+1}$ and $P(\theta) = \infty$, $\forall \theta$ outside the feasible range;

ii) $P(\theta)$ continuos in the feasible range.

Next, define the penalized least squares cost function

$$D_t(\theta) := V_t(\theta) + P(\theta). \quad (2)$$

The parameter estimate obtained by minimizing (2) will be denoted by $\hat{\theta}_t$.

Penalized techniques are well known in the field of operation research (see *e.g.* [5]). The result of Theorem 2 below makes the use of penalized techniques very attractive in the area of adaptive control (due to space limitations we omit all the proofs).

Theorem 2 (*properties of the estimates*)
$\hat{\theta}_t$ belongs to a closed set strictly contained in the feasible range for any t. Moreover, partial consistency holds, *i.e.* denoting by $\hat{\theta}_{E,t}$ the component of $\hat{\theta}_t$ along the excitation subspace, $\lim_{t\to\infty} \hat{\theta}_{E,t} = \theta^o_E$ almost surely. □

2 Self-tuning control

In the literature many control techniques have been proposed which exhibit stabilizing properties when applied to invariant plants. Among them, well-known methods ensure stability subject only to the assumption that the system is controllable (*e.g.* infinite-horizon LQ control [6], pole-placement [7], receding-horizon control [8]). When applying these techniques in an adaptive fashion, according to the certainty equivalence principle, one regards the estimated model as if it were the actual plant. Therefore, lack of controllability of such a model leads to a paralysis in the control selection (see *e.g.* [4] and [9]).

This suggests the implementation of adaptive control schemes with identification based on penalized LS rather than using standard LS. Since one of the major issue is to guarantee controllability in the asymptotically identified model, a wise choice of the penalty term $P(\theta)$ in (2) is

$$P(\theta) = \frac{1}{|Sylv(\theta)|} + P_1(\theta), \quad (3)$$

where $Sylv(\theta)$ is the *Sylvester resultant* of polynomials $q^{\bullet}A(\theta;q^{-1})$ and $q^{\bullet-d}B(\theta;q^{-1})$ (see *e.g.* [10]). It is introduced so as to avoid parameterizations which correspond to uncontrollable (or nearly uncontrollable) systems. As for $P_1(\theta)$, its role is to take into account possible additional constraints on the parameterization. As Theorem 2 states, the cost function (2) with penalty (3) still guarantees the partial consistency property. Thanks to this, one can prove the following results.

Theorem 3 (*L^2-stability*)
Consider system (1) adaptively regulated in a certainty equivalent fashion by means of any control technique able to stabilize a known controllable plant. If the estimate of the true parameter θ^o is obtained through algorithm (2) with $P(\theta)$ given by (3) such that $P(\theta^o) < \infty$, then the adaptively controlled system is L^2-stable:

$$\limsup_{N\to\infty} \frac{1}{N}\sum_{t=1}^{N}[y_t^2 + u_t^2] < \infty \text{ almost surely.} \quad \square$$

Theorem 4 (*optimality*)
Under the same assumptions as in Theorem 3, the adaptively controlled system achieves asymptotic optimality in the sense that its behavior in the long run tends to that of the *estimated* control system. □

References

[1] S. Bittanti, P. Bolzern, and M. Campi, "Recursive least squares identification algorithms with incomplete excitation: convergence analysis and application to adaptive control", *IEEE Trans. on Automatic Control*, AC-35, 1990, pp. 1371-1373.

[2] H. Rootzen and J. Sternby, "Consistency in least-squares estimation: a bayesian approach", *Automatica*, 20, 1984, pp. 471-475.

[3] S. Bittanti and M. Campi, "Least squares based self-tuning control systems", *Identification, Adaptation, Learning - The science of learning models from data* (S.Bittanti and G.Picci eds.), Springer-Verlag NATO ASI series - Computer and systems sciences, 1996, pp. 339-365.

[4] M.C. Campi, "The problem of pole-zero cancellation in transfer function identification and application to adaptive stabilization", *Automatica*, 32, 1996, pp. 849-857.

[5] D.G. Luenberger, *Linear and Nonlinear Programming*, Addison-Wesley, 2nd edition, 1984.

[6] B.D.O. Anderson and J.B. Moore, *Optimal Control - Linear Quadratic Methods*. Prentice Hall, 1989.

[7] K. Aström and B. Wittenmark, "Self-tuning controllers based on pole-zero placement", In *IEE Proceedings*, vol.127 D, 1980, pp. 120-130.

[8] E. Mosca and J. Zhang, "Stable redesign of predictive control", *Automatica*, 28, 1992, pp. 1229-1233.

[9] R. Lozano and X.H. Zhao, "Adaptive pole placement without excitation probing signals", *IEEE Trans. on Automatic Control*, AC-39, 1994, pp. 47-58.

[10] T. Kailath, *Linear Systems Theory*, Englewood Cliffs, NJ: Prentice-Hall, 1980.