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DO SELF-TUNING CONTROL SYSTEMS TUNE?

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Abstract: Yes, provided that a suitable notion of tuning is taken.

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1. TUNING AND IDEAL TUNING OF SELF-TUNING CONTROL SYSTEMS

1.1 Mathematical framework

Consider the noise-free SISO plant

$$\mathbf{P}: \quad A(\vartheta^\circ;q^{-1})y(t) = B(\vartheta^\circ;q^{-1})u(t-d),$$

where

$$A(\vartheta^\circ;q^{-1}) = 1 - \sum_{i=1}^{n} a_i^\circ q^{-i},$$

$$B(\vartheta^\circ;q^{-1}) = \sum_{i=0}^{m} b_i^\circ q^{-i}, \qquad b_0^\circ \neq 0, \quad d \geq 1.$$

Here, $u(\cdot)$ is the control variable, $y(\cdot)$ the plant output and

$$\vartheta^\circ = [a_1^\circ \; a_2^\circ \; \cdots \; a_n^\circ \; b_0^\circ \; b_1^\circ \; \cdots \; b_m^\circ]^T$$

is the plant parameter vector.

Plant $\mathbf{P}$ is initialized at time $t=1$ with deterministic values $y(0)$, $y(-1), \ldots, y(-n+1)$, $u(0)$, $u(-1)$, $\ldots$, $u(-d-m+1)$.

We assume that $q^n A(\vartheta^\circ;q^{-1})$ and $q^m B(\vartheta^\circ;q^{-1})$ are coprime.

The main motivation for adaptive control is the lack of knowledge on some characteristics of the plant. Herein, the following hypothesis is made: the structure of the equation is known, i.e. d, m and n are known, but the true parameter vector ϑ° is not available. Correspondingly, one can resort to an identification algorithm to get a model of the plant.

Consider the family of (deterministic) ARX models parameterized in the vector $\vartheta = [a_1 \; a_2 \; \cdots \; a_n \; b_0 \; b_1 \; \cdots \; b_m]^T$:

$$\mathbf{M}(\vartheta): \quad A(\vartheta;q^{-1})y(t) = B(\vartheta;q^{-1})u(t-d), \qquad (1)$$

with

$$A(\vartheta;q^{-1}) = 1 - \sum_{i=1}^{n} a_i q^{-i},$$

$$B(\vartheta;q^{-1}) = \sum_{i=0}^{m} b_i q^{-i}.$$

Then, denoting by $\hat{\vartheta}(t)$ the parameter estimate at time t, the estimated model is given by $\mathbf{M}(\hat{\vartheta}(t))$.

In this paper, we assume that estimate $\hat{\vartheta}(t)$ is computed through the Recursive Least Squares (RLS) identification algorithm. Rewrite model (1) as:

$$\mathbf{M}(\vartheta): \quad y(t) = \varphi(t-1)^T\vartheta,$$

where the observation vector $\varphi(t-1)$ is given by

$$\varphi(t-1) = [y(t-1)\ y(t-2)\ \cdots\ y(t-n)$$
$$u(t-d)\ u(t-d-1)\ \cdots\ u(t-d-m)]^T.$$

Then, $\hat{\vartheta}(t)$ is recursively computed by means of the equations:

$$\hat{\vartheta}(t) = \hat{\vartheta}(t-1) + P(t)\varphi(t-1)\left[y(t) - \varphi(t-1)^T\hat{\vartheta}(t-1)\right],$$
$$\hat{\vartheta}(0) = \hat{\vartheta}_o$$

$$P(t) = P(t-1) - \frac{P(t-1)\varphi(t-1)\varphi(t-1)^T P(t-1)}{1 + \varphi(t-1)^T P(t-1)\varphi(t-1)},$$
$$P(0) = P(0)^T > 0.$$

We finally introduce a general linear control law for model $M(\vartheta)$. This is defined by the equation:

$$C(\vartheta):\quad u(t) = R(\vartheta;q^{-1})u(t) + S(\vartheta;q^{-1})y(t) + T(\vartheta;q^{-1})y^*(t),$$

with

$$R(\vartheta;q^{-1}) = \sum_{i=1}^{\alpha} r_i(\vartheta)q^{-i},$$

$$S(\vartheta;q^{-1}) = \sum_{i=0}^{\beta} s_i(\vartheta)q^{-i},$$

$$T(\vartheta;q^{-1}) = \sum_{i=-\gamma}^{\delta} t_i(\vartheta)q^{-i},$$

$y^*(\cdot)$ being a bounded reference signal.

The above control law is general indeed, and encompasses as special cases all the most popular techniques which have been proposed in the literature. Among others, we mention: infinite-horizon LQ control (Anderson and Moore, 1989); pole-placement (Aström and Wittenmark, 1980); receding-horizon control (Mosca and Zhang, 1992) and (Chisci and Mosca, 1993).

We will conform to the certainty equivalence principle, which amounts to take $C(\hat{\vartheta}(t))$ as present control law for the true plant. Therefore, the *real system*, that is the true plant together with the actual control law (see Fig.1) is given by the equations:

$$\Sigma(\vartheta^\circ,\hat{\vartheta}(t)) \begin{cases} A(\vartheta^\circ;q^{-1})y(t) = B(\vartheta^\circ;q^{-1})u(t-d) \\ u(t) = R(\hat{\vartheta}(t);q^{-1})u(t) + S(\hat{\vartheta}(t);q^{-1})y(t) \\ \quad + T(\hat{\vartheta}(t);q^{-1})y^*(t). \end{cases}$$

1.2 *Ideal tuning and tuning*

The original hope of adaptive control was that the self adjustment of the controller would eventually result in the same performance as that achievable if the true plant were known. In other words, real system

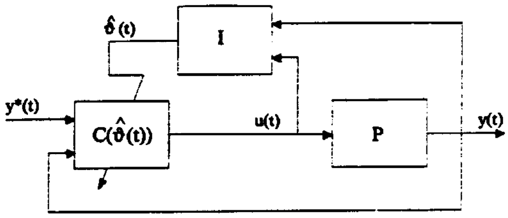

Figure 1. Real System

$\Sigma(\vartheta^\circ,\hat{\vartheta}(t))$ was hoped to behave in the long run as the *ideal system* precisely given below (Fig.2):

$$\Sigma(\vartheta^\circ,\vartheta^\circ) \begin{cases} A(\vartheta^\circ;q^{-1})y(t) = B(\vartheta^\circ;q^{-1})u(t-d) \\ u(t) = R(\vartheta^\circ;q^{-1})u(t) + S(\vartheta^\circ;q^{-1})y(t) \\ \quad + T(\vartheta^\circ;q^{-1})y^*(t). \end{cases}$$

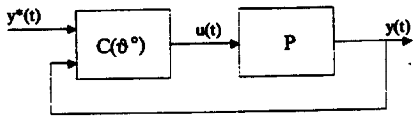

Figure 2. Ideal System

Denoting by $W_1^\circ(q^{-1})$ the transfer function between y^* and y, this leads to the following definition:

Definition 1 (Ideal tuning)
Real system $\Sigma(\vartheta^\circ,\hat{\vartheta}(t))$ is said to be *ideally tuning* if

$$\lim_{t\to\infty}\left(y(t) - W_1^\circ(q^{-1})y^*(t)\right) = 0. \quad ■$$

Ideal tuning can indeed be achieved in some special cases such as minimum variance (Goodwin *et al.* 1981; Bittanti *et al.*, 1992) and pole-placement (van Schuppen, 1994). However, it is also well known that ideal tuning does not hold in general. Then, the question arises whether it is possible to set a general theory of tuning (which holds true independently of the adopted control law) by resorting to a weaker notion of tuning. This is precisely formalized in the following.

At time t, according to the certainty equivalence principle, model $M(\hat{\vartheta}(t))$ is regarded as if it were the actual plant and the controller calibrates its parameters so as to achieve a desired behavior for such a model. Therefore, the so-called *imaginary system* (Fig.3):

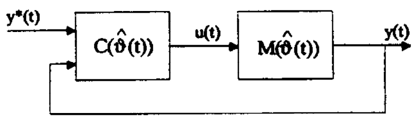

Figure 3. Imaginary System

$$\Sigma(\hat{\vartheta}(t),\hat{\vartheta}(t)) \begin{cases} A(\hat{\vartheta}(t);q^{-1})y(t) = B(\hat{\vartheta}(t);q^{-1})u(t-d) \\ u(t) = R(\hat{\vartheta}(t);q^{-1})u(t) + S(\hat{\vartheta}(t);q^{-1})y(t) \\ \quad + T(\hat{\vartheta}(t);q^{-1})y^{*}(t), \end{cases}$$

represents the *desired behavior* at time t. If we push our luck and assume for a moment that the estimate converges, say $\hat{\vartheta}(t) \to \hat{\vartheta}(\infty)$, then the *imaginary system* would tend to the following time-invariant system (*asymptotic imaginary system*, Fig.4):

$$\Sigma(\hat{\vartheta}(\infty),\hat{\vartheta}(\infty)) \begin{cases} A(\hat{\vartheta}(\infty);q^{-1})y(t) = B(\hat{\vartheta}(\infty);q^{-1})u(t-d) \\ u(t) = R(\hat{\vartheta}(\infty);q^{-1})u(t) + S(\hat{\vartheta}(\infty);q^{-1})y(t) \\ \quad + T(\hat{\vartheta}(\infty);q^{-1})y^{*}(t), \end{cases}$$

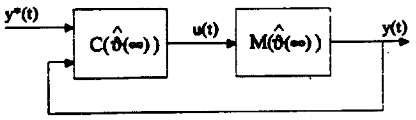

Figure 4. Asymptotic Imaginary System

We say that the adaptive control system is tuning if its behavior resembled the one of the asymptotic imaginary system:

Definition 2 (Tuning)
Real system $\Sigma(\vartheta^{\circ},\hat{\vartheta}(t))$ is said to be *tuning* if

$$\lim_{t\to\infty}\left(y(t) - W_1(q^{-1})y^{*}(t)\right) = 0,$$

where

$$W_1(q^{-1}) = \frac{B(\hat{\vartheta}(\infty);q^{-1})q^{-d}T(\hat{\vartheta}(\infty);q^{-1})}{A(\hat{\vartheta}(\infty);q^{-1})\left[1-R(\hat{\vartheta}(\infty);q^{-1})\right] - B(\hat{\vartheta}(\infty);q^{-1})q^{-d}S(\hat{\vartheta}(\infty);q^{-1})}$$

is the transfer function between y^{*} and y in the asymptotic imaginary system. ■

In this paper we provide a unitary approach which permits to analyze any adaptive control systems under reasonable assumptions. Our main result is that, differently from ideal tuning, tuning holds in general without extra assumptions on the actual system under control such as its stability or the minimum-phase condition.

1.3 *A bibliographical note*

In this paper, we only study the tuning property in the mathematically elementary case in which the system is noise-free. Moreover, the assumption that the asymptotic imaginary system is controllable will be made (see Sect.1.4). It is important to note that this standpoint is taken in the present contribution only for the sake of simplicity and clarity of exposition. As a matter of fact, a full treatment of the matter is now available (even though only partially published) and can be found in the contributions listed below.

A complete analysis of the tuning property in a deterministic context with examples and a tutorial style is provided in (Bittanti and Campi, 1995). The noisy case is dealt with in (Campi, 1994) still under the assumption that the asymptotic imaginary system is controllable. The controllability problem of the asymptotic imaginary system is the subject of many papers. Among them, those that are more closely related to the approach assumed in this contribution are (Lozano and Zhao, 1994) in a deterministic setting and (Campi, 1996) in a stochastic framework.

1.4 *Results*

Assume that the following assumptions hold true.

Assumption 1
The controller parameters $r_i(\vartheta)$, $s_i(\vartheta)$ and $t_i(\vartheta)$ are continuous in $\hat{\vartheta}(\infty)$. ■

Assumption 2
The asymptotic imaginary system $\Sigma(\hat{\vartheta}(\infty),\hat{\vartheta}(\infty))$ is stable. ■

Remark
Many well established techniques ensure asymptotic stability and continuity of the controller parameters when applied to a known time-invariant plant under the sole condition that the plant is controllable. This is the case for LQ control, pole-placement, receding-horizon control, to quote but a few. ■

In this section we state the main results of this contribution, the proofs of which are provided in the next section.

First, the RLS algorithm provides estimates which are generally convergent. The asymptotic estimate is or is not coincident with the true parameterization depending on the excitation characteristics of signals. This constitutes a comfortable starting point for the forthcoming analysis.

Definition 3 (Excitation and unexcitation subspaces - (Bittanti et al., 1990))
The subspace $\bar{\mathcal{E}} = \left\{ x \in \mathbb{R}^{n+m+1} \mid x^{T}\sum_{\tau=1}^{\infty}\varphi(\tau-1)\varphi(\tau-1)^{T}x < \infty \right\}$ is named *unexcitation subspace*.

Its orthogonal complement $\mathcal{E} = \bar{\mathcal{E}}^{\perp}$ is named *excitation subspace*. ∎

Theorem 1 (RLS properties)
The RLS estimate $\hat{\vartheta}(t)$ is asymptotically convergent:

$$\lim_{t\to\infty} \hat{\vartheta}(t) = \hat{\vartheta}(\infty).$$

Moreover, denoting by $\hat{\vartheta}_E(\infty)$ and ϑ°_E the projections of $\hat{\vartheta}(\infty)$ and ϑ° onto the excitation subspace $\mathcal{E}$, it turns out that

$$\hat{\vartheta}_E(\infty) = \vartheta^\circ_E.$$ ∎

Under Assumptions 1 and 2, signals $u(\cdot)$ and $y(\cdot)$ keeps bounded.

Theorem 2 (Stability)
If Assumptions 1 and 2 are met with, then there exists a constant c such that

$$|u(t)| < c \text{ and } |y(t)| < c, \quad \forall t.$$ ∎

Finally, under the same assumptions, the tuning property holds in general.

Theorem 3 (Tuning)
If Assumptions 1 and 2 are met with, then the real system $\Sigma(\vartheta^\circ, \hat{\vartheta}(t))$ is tuning according to Definition 2. ∎

We emphasize that Theorem 3 guarantees that the adaptively controlled true system (*real system*) behaves closely to the nonadaptively controlled asymptotically identified system (*asymptotic imaginary system*). In no way, this conclusion entails that the performance of the real system is close to the one which would be obtained if the real plant were actually known (ideal tuning).

The above set of theorems provides a unitary theory which can be applied to any RLS-based adaptive scheme regardless of the control law used and with no extra assumption on the true system such as the minimum-phase condition.

2. PROOFS

Proof of Theorem 1
Introduce the parameter error

$$\tilde{\vartheta}(t) = \hat{\vartheta}(t) - \vartheta^\circ.$$

The following recursion for $\tilde{\vartheta}(t)$ is easily derived

$$\tilde{\vartheta}(t) = \tilde{\vartheta}(t-1) - P(t)\varphi(t-1)\varphi(t-1)^T\tilde{\vartheta}(t-1). \quad (2)$$

Then, by noting that the recursion for $P(t)^{-1}$ is

$$P(t)^{-1} = P(t-1)^{-1} + \varphi(t-1)\varphi(t-1)^T, \quad (3)$$

we get

$$\begin{aligned} P(t)^{-1}\,\tilde{\vartheta}(t) &= P(t)^{-1}\,\tilde{\vartheta}(t-1) - \varphi(t-1)\varphi(t-1)^T\tilde{\vartheta}(t-1) \\ &= P(t-1)^{-1}\,\tilde{\vartheta}(t-1), \end{aligned}$$

which corresponds to

$$P(t)^{-1}\left[\hat{\vartheta}(t) - \vartheta^\circ\right] = \text{const}.$$

Since $P(t)$ converges, this obviously implies that $\hat{\vartheta}(t)$ is convergent to some limit $\hat{\vartheta}(\infty)$.

Introduce now the Lyapunov-like function

$$V(t) = \tilde{\vartheta}(t)^T P(t)^{-1}\tilde{\vartheta}(t).$$

From (2) and (3), the recursive expression for $V(t)$ can be worked out:

$$V(t) = V(t-1) - \frac{[\varphi(t-1)^T\tilde{\vartheta}(t-1)]^2}{1 + \varphi(t-1)^T P(t-1)\varphi(t-1)}.$$

This implies that function $V(t)$ is monotonically decreasing, so that

$$V(0) \ge V(t).$$

By observing that $P(t)^{-1} = P(0)^{-1} + \sum_{\tau=1}^{t} \varphi(\tau-1)\varphi(\tau-1)^T$, one obtains

$$V(0) \ge \tilde{\vartheta}(t)^T M(t)\tilde{\vartheta}(t),$$

where

$$M(t) = \sum_{\tau=1}^{t} \varphi(\tau-1)\varphi(\tau-1)^T.$$

From this

$$\lim_{t\to\infty} \tilde{\vartheta}(\infty)^T M(t)\tilde{\vartheta}(\infty) < \infty, \quad (4)$$

where

$$\tilde{\vartheta}(\infty) = \hat{\vartheta}(\infty) - \vartheta^\circ.$$

Partition now $\tilde{\vartheta}(\infty)$ in the excitation and unexcitation components: $\tilde{\vartheta}(\infty) = \tilde{\vartheta}_E(\infty) + \tilde{\vartheta}_U(\infty)$ (where $\tilde{\vartheta}_E(\infty)\in\mathcal{E}$ and $\tilde{\vartheta}_U(\infty)\in\bar{\mathcal{E}}$). Then, from (4) we obtain ($\varphi_E(\tau-1)$ [$\varphi_U(\tau-1)$] denotes projection of $\varphi(\tau-1)$ onto $\mathcal{E}$ [$\bar{\mathcal{E}}$]):

$$\begin{aligned} \infty &> \lim_{t\to\infty} \tilde{\vartheta}(\infty)^T M(t)\tilde{\vartheta}(\infty) \\ &= \lim_{t\to\infty} \sum_{\tau=1}^{t} \left(\varphi(\tau-1)^T\tilde{\vartheta}(\infty)\right)^2 \\ &= \lim_{t\to\infty} \sum_{\tau=1}^{t} \left(\varphi_E(\tau-1)^T\tilde{\vartheta}_E(\infty) + \varphi_U(\tau-1)^T\tilde{\vartheta}_U(\infty)\right)^2 \end{aligned}$$

$$\geq \lim_{t\to\infty} \left\{ \tilde{\vartheta}_E(\infty)^T \left(\sum_{\tau=1}^{t} \varphi_E(\tau-1)\varphi_E(\tau-1)^T \right) \tilde{\vartheta}_E(\infty) \right.$$

$$+ \tilde{\vartheta}_U(\infty)^T \left(\sum_{\tau=1}^{t} \varphi_U(\tau-1)\varphi_U(\tau-1)^T \right) \tilde{\vartheta}_U(\infty)$$

$$- 2 \left[\tilde{\vartheta}_E(\infty)^T \left(\sum_{\tau=1}^{t} \varphi_E(\tau-1)\varphi_E(\tau-1)^T \right) \tilde{\vartheta}_E(\infty) \right]^{1/2} \times$$

$$\left. \times \left[\tilde{\vartheta}_U(\infty)^T \left(\sum_{\tau=1}^{t} \varphi_U(\tau-1)\varphi_U(\tau-1)^T \right) \tilde{\vartheta}_U(\infty) \right]^{1/2} \right\}.$$

In view of the very definition of excitation subspace, this inequality entails that $\tilde{\vartheta}_E(\infty) = 0$. ■

Proof of Theorem 2

Set

$$e(t) = \varphi(t-1)^T(\vartheta^\circ - \hat{\vartheta}(t)).$$

Then, the time evolution of vector $z(t) = [y(t)\ u(t)]^T$ generated by $\Sigma(\vartheta^\circ;\hat{\vartheta}(t))$ is given by the equation

$$z(t) = D(\hat{\vartheta}(t);q^{-1})z(t-1) + \begin{bmatrix} e(t) \\ T(\hat{\vartheta}(t);q^{-1})y^*(t) \end{bmatrix}, \qquad (5)$$

where matrix

$$D(\hat{\vartheta}(t);q^{-1}) = \begin{bmatrix} \left[1 - A(\hat{\vartheta}(t);q^{-1})\right]q & B(\hat{\vartheta}(t);q^{-1})q^{-d+1} \\ S(\hat{\vartheta}(t);q^{-1})q & R(\hat{\vartheta}(t);q^{-1})q \end{bmatrix}$$

describes the dynamics of $\Sigma(\hat{\vartheta}(t);\hat{\vartheta}(t))$.

Since the asymptotic imaginary system is stable (Assumption 2), the movement of the autonomous system $z(t) = D(\hat{\vartheta}(\infty);q^{-1})z(t-1)$ initialized at time t_0 tends exponentially to zero: $\|\bar{z}(t)\| \leq \alpha\rho^{\Delta t}\|\bar{z}(t_0)\|$, $\rho<1$, $\Delta t = t - t_0$, where $\bar{z}(t) = [y(t)\ y(t-1)\ \cdots\ y(t+1-\max\{n,\beta\})\ u(t)\ u(t-1)\ \cdots\ u(t+1-\max\{d+m,\alpha\})]^T$. Taking into account that $\hat{\vartheta}(t) \to \hat{\vartheta}(\infty)$ and $D(\vartheta;q^{-1})$ is continuous in $\hat{\vartheta}(\infty)$, for the autonomous *imaginary system* $(z(t) = D(\hat{\vartheta}(t);q^{-1})z(t-1))$ we then have $\|\bar{z}(t)\| \leq \alpha\rho^{\Delta t}\|\bar{z}(t_0)\| + \gamma(t_0,\Delta t)\|\bar{z}(t_0)\|$, where $\gamma(\cdot,\cdot)$ is such that $\gamma(t_0,\Delta t)\to 0$, $t_0\uparrow\infty$, Δt fixed. Choose $\Delta\bar{t}$ such that $\alpha\rho^{\Delta\bar{t}} = \epsilon < 1$ and $\bar{t}_0$ such that $|\gamma(t_0,\Delta\bar{t})| \leq \delta < 1-\epsilon$, $\forall t_0 \geq \bar{t}_0$. Then, $\forall t_0 \geq \bar{t}_0$, one has $\|\bar{z}(t_0+\Delta\bar{t})\| \leq (\epsilon+\delta)\|\bar{z}(t_0)\|$, from which the uniform exponential stability of the autonomous *imaginary system* follows. Turn now to consider the *real system*. In view of the stability of the autonomous *imaginary system* and representation (5) of $\Sigma(\vartheta^\circ;\hat{\vartheta}(t))$, $z(t)$ generated by the *real system* can be bounded as follows:

$$\|z(t)\| \leq c_1 + c_2 \sum_{\tau=1}^{t} \nu^{t-\tau} \left\| \begin{bmatrix} e(\tau) \\ T(\hat{\vartheta}(\tau);q^{-1})y^*(\tau) \end{bmatrix} \right\|$$

$$\leq c_1 + c_2 \sum_{\tau=1}^{t} \nu^{t-\tau} |\varphi(\tau-1)^T(\vartheta^\circ - \hat{\vartheta}(\tau))|$$

$$+ c_2 \sum_{\tau=1}^{t} \nu^{t-\tau} |T(\hat{\vartheta}(\tau);q^{-1})y^*(\tau)|, \qquad (6)$$

c_1, c_2 and $\nu<1$ being suitable constants. By virtue of the boundedness of the reference output, the term $c_2 \sum_{\tau=1}^{t} \nu^{t-\tau}|T(\hat{\vartheta}(\tau);q^{-1})y^*(\tau)|$ turns out to be itself bounded. As for the second term on the right hand side it can be handled as follows:

$$c_2 \sum_{\tau=1}^{t} \nu^{t-\tau} |\varphi(\tau-1)^T(\vartheta^\circ - \hat{\vartheta}(\tau))|$$

$$\leq c_2 \sum_{\tau=1}^{t} \nu^{t-\tau} \left\{ |\varphi_E(\tau-1)^T(\vartheta_E^\circ - \hat{\vartheta}_E(\tau))| + |\varphi_U(\tau-1)^T(\vartheta_U^\circ - \hat{\vartheta}_U(\tau))| \right\}$$

$$\leq c_2 \sum_{\tau=1}^{t} \nu^{t-\tau} \|\varphi_E(\tau-1)\| \|\vartheta_E^\circ - \hat{\vartheta}_E(\tau)\|$$

$$+ \quad c_2 \sum_{\tau=1}^{t} \nu^{t-\tau} \|\varphi_U(\tau-1)\| \|\vartheta_U^\circ - \hat{\vartheta}_U(\tau)\|.$$

The second term in this last expression tends to zero because of the boundedness of $\|\vartheta_U^\circ - \hat{\vartheta}_U(\tau)\|$ and the fact that $\varphi_U(\tau-1) \to 0$ as $\tau \to \infty$ (see the definition of unexcitation subspace). Therefore, bearing in mind the definition of observation vector, from (6) we finally get

$$\|\varphi(t)\| \leq c_3 + c_4 \sum_{\tau=1}^{t} \nu^{t-\tau} \|\varphi_E(\tau-1)\| \|\vartheta_E^\circ - \hat{\vartheta}_E(\tau)\|$$

$$\leq c_3 + c_4 \sum_{\tau=1}^{t} \nu^{t-\tau} \|\varphi(\tau-1)\| \|\vartheta_E^\circ - \hat{\vartheta}_E(\tau)\|,$$

being c_3 and c_4 suitable constants.

Since $\vartheta_E^\circ - \hat{\vartheta}_E(t) \to 0$, this inequality implies that $\|\varphi(t)\|$ remains bounded, from which the thesis immediately follows. ■

Proof of Theorem 3

By virtue of Assumption 3, polynomial $A(\hat{\vartheta}(\infty);q^{-1})\left[1 - R(\hat{\vartheta}(\infty);q^{-1})\right] - B(\hat{\vartheta}(\infty);q^{-1})q^{-d}S(\hat{\vartheta}(\infty);q^{-1})$ is Hurwitz, so that from the fact that $\varphi(t-1)^T(\vartheta^\circ - \hat{\vartheta}(t)) \to 0$, we have

$$\frac{1 - R(\hat{\vartheta}(\infty);q^{-1})}{A(\hat{\vartheta}(\infty);q^{-1})\left[1 - R(\hat{\vartheta}(\infty);q^{-1})\right] - B(\hat{\vartheta}(\infty);q^{-1})q^{-d}S(\hat{\vartheta}(\infty);q^{-1})} \times \varphi(t-1)^T(\vartheta^\circ - \hat{\vartheta}(t))$$

$$= \frac{1 - R(\hat{\vartheta}(\infty);q^{-1})}{A(\hat{\vartheta}(\infty);q^{-1})\left[1-R(\hat{\vartheta}(\infty);q^{-1})\right] - B(\hat{\vartheta}(\infty);q^{-1})q^{-d}S(\hat{\vartheta}(\infty);q^{-1})}$$

$$\times \left\{y(t) - \left[1 - A(\hat{\vartheta}(t);q^{-1})\right]y(t) - B(\hat{\vartheta}(t);q^{-1})q^{-d}u(t)\right\}$$

$$\to 0.$$

Since $\hat{\vartheta}(t) \to \hat{\vartheta}(\infty)$ and $|y(t)|$ is bounded (Theorem 2), we can replace $A(\hat{\vartheta}(t);q^{-1})y(t)$ in the previous expression with $A(\hat{\vartheta}(\infty);q^{-1})y(t)$. For an analogous reason, in place of $[1 - R(\hat{\vartheta}(\infty);q^{-1})]B(\hat{\vartheta}(t);q^{-1})q^{-d}u(t)$, we can write $B(\hat{\vartheta}(t);q^{-1})[1 - R(\hat{\vartheta}(t-d);q^{-1})]q^{-d}u(t)$, which equals $B(\hat{\vartheta}(t);q^{-1})[S(\hat{\vartheta}(t-d);q^{-1})y(t-d) + T(\hat{\vartheta}(t-d);q^{-1})y^{*}(t-d)]$ and this last expression can be finally replaced by $B(\hat{\vartheta}(\infty);q^{-1})[S(\hat{\vartheta}(\infty);q^{-1})y(t-d) + T(\hat{\vartheta}(\infty);q^{-1})y^{*}(t-d)]$. In conclusion, we have

$$y(t) - \frac{B(\hat{\vartheta}(\infty);q^{-1})q^{-d}T(\hat{\vartheta}(\infty);q^{-1})\ y^{*}(t)}{A(\hat{\vartheta}(\infty);q^{-1})\left[1-R(\hat{\vartheta}(\infty);q^{-1})\right] - B(\hat{\vartheta}(\infty);q^{-1})q^{-d}S(\hat{\vartheta}(\infty);q^{-1})}$$

$$\to 0,$$

that is the self-optimality condition. ■

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