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A new recursive identification algorithm for singularity free adaptive control

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Abstract

Reportedly, guaranteeing the controllability of the estimated system is a crucial problem in adaptive control. In this paper, we introduce a least-squares-based identification algorithm for stochastic SISO systems, which secures the uniform controllability of the estimated system and presents closed-loop identification properties similar to those of the least-squares algorithm. The proposed algorithm is recursive and, therefore, easily implementable. Its use, however, is confined to cases in which the parameter uncertainly is highly structured. © 1998 Elsevier Science B.V. All rights reserved.

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1. Introduction

It is well known [1,2,4–6,10,13–15,17,18] that the possible occurrence of pole-zero cancellations in the estimated model hampers the development of adaptive control algorithms for possibly nonminimum-phase systems. As a matter of fact, many well-established stability and performance results exist which are applicable under a uniform controllability assumption of the estimated model (see e.g. [3,20,21]). On the other hand, however, standard identification algorithms do not guarantee such a controllability property in the absence of suitable persistence of excitation conditions, as it turns out to be often the case in closed-loop operating conditions.

Many contributions have appeared in the literature over the last decade to address the controllability problem in adaptive control. A first approach consists in the a posteriori modification of the least-squares estimate [4,5,13,14]. By exploiting the properties of the least-squares covariance matrix, these methods secure controllability, while preserving the valuable closed-loop properties of the least-squares identification algorithm. The main drawback of this approach is that its computational complexity highly increases with the order of the system (see [14]). Therefore, an on-line implementation of these methods turns out to be generally impossible. A second approach [10, 15, 17] directly modifies the identification algorithm so as to force the estimate to belong to an a priori known region containing the true parameter and such that all the models in that region are controllable. These methods lead to easily implementable algorithms, but they are suitable for systems subject to bounded noise only. The design of recursive (and, therefore, on-line-implementable) identification methods able to guarantee the model controllability in the case of stochastic

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unbounded noise is still an open problem at the present stage of the research. A further class of approaches is characterized by the design of the controller according to a method different from the standard certainty equivalence strategy. In [11], an overparameterized representation of the plant and the controller is used to define a control scheme taking the form of an adaptively tuned linear controller with an additional nonlinear time-varying feedback signal. In [1], instead, the system is reparameterized in the form of an approximate model which is controllable regardless of the numerical value of the parameters. Finally, it is worth mentioning the interesting strategy introduced in [16, 18] which adopts a logic-based switching controller for solving the controllability problem. All these alternatives to the standard certainty equivalence strategy, however, deal with the case when the system is deterministic, i.e. when there is no stochastic noise acting on the system.

In this paper, we introduce a recursive least-squares-based identification algorithm for systems subject to stochastic white noise which ensures the uniform controllability of the estimated model. Our method is applicable under a stringent condition concerning the a priori knowledge on the uncertainty region to which the true parameter belongs (see Assumption 3 in Section 2). Such a condition may or may not be satisfied depending on the application at hand. In the case such a knowledge is in fact available, our identification algorithm represents an efficient and easily implementable method to circumvent the controllability problem. Moreover, as we shall show in Section 3, our identification algorithm retains the closed-loop identification properties of the standard least-squares method (Theorem 2 in Section 3). This is of crucial importance in adaptive control applications (see e.g. [7, 8]). In this regard, the interested reader is referred to contribution [19], where a stability result is worked out for adaptive pole placement on the basis of such properties.

2. The system and the uncertainty region

We consider a discrete-time stochastic SISO system described by the following ARX model:

$$A(\vartheta^\circ; q^{-1})\, y_t = B(\vartheta^\circ; q^{-1})\, u_t + n_t, \tag{1}$$

where $A(\vartheta^\circ; q^{-1})$ and $B(\vartheta^\circ; q^{-1})$ are polynomials in the unit-delay operator q^{-1} depending on the system parameter vector $\vartheta^\circ = [a_1^\circ\, a_2^\circ \dots a_n^\circ\, b_d^\circ\, b_{d+1}^\circ \dots b_{d+m}^\circ]^{\mathrm{T}}$. Precisely, they are given by

$$A(\vartheta^\circ; q^{-1}) = 1 - \sum_{i=1}^{n} a_i^\circ q^{-i}$$

and

$$B(\vartheta^\circ; q^{-1}) = \sum_{i=d}^{d+m} b_i^\circ q^{-i}.$$

We make the assumption that $n > 0$ and $m > 0$, since if $n = 0$ or $m = 0$ the controllability issue automatically disappears. As for the stochastic disturbance process $\{n_t\}$, it is described as a martingale difference sequence with respect to a filtration $\{\mathscr{F}_t\}$, satisfying the following conditions:

(A.1) $\sup_t E[|n_{t+1}|^\beta / \mathscr{F}_t] < \infty$, almost surely for some $\beta > 2$,

(A.2) $\liminf_{t\to\infty} (1/t) \sum_{k=1}^{t} n_k^2 > 0$.

In this paper, a new identification algorithm for system (1) is introduced, which secures the estimated model controllability, while preserving the least squares algorithm closed-loop identification properties. These results are worked out under the assumption that the following a priori knowledge is available:

(A.3) ϑ° is an interior point of $S(\bar{\vartheta}, r) = \{\vartheta \in R^{n+m+1}: \|\vartheta - \bar{\vartheta}\| \leqslant r\}$, where the $n+m+1$-dimensional sphere $S(\bar{\vartheta}, r)$ is such that all models with parameter $\vartheta \in S(\bar{\vartheta}, r)$ are controllable.

Assumption (A.3) is certainly a stringent condition. It requires that the a priori parameter uncertainty is restricted enough so that the uncertainty region can be described as a sphere completely embedded in the controllability region. In this connection, the center $\bar{\vartheta}$ of the sphere should be thought of as a nominal, a priori known, value of the uncertain parameter ϑ°, obtained either by physical knowledge of the plant or by some coarse off-line identification procedure. The identification algorithm should then be used to refine the parameter estimate during the normal on-line operating condition of the control system so as to better tune the controller to the actual plant characteristics.

3. The recursive identification algorithm

Letting

$$\varphi_t = [y_t \dots y_{t-(n-1)}\, u_{t-(d-1)} \dots u_{t-(d+m-1)}]^{\mathrm{T}} \tag{2}$$

be the observation vector, system (1) can be given the usual regression-like form

$$y_t = \varphi_{t-1}^{\mathrm{T}} \vartheta^\circ + n_t. \tag{3}$$

The recursive algorithm for the estimation of parameter ϑ° is given by the following recursive procedure (see below for an interpretation of this procedure):

1. Compute P_t according to the following steps:
   set $T_0 = P_{t-1}$;
   for $i = 1$ to $n + m + 1$, set

$$\begin{array}{l} \qquad\quad i \\ \qquad\quad \downarrow \\ \phi_i = [0 \dots 1 \dots 0]^{\mathrm{T}} \end{array}$$

   and compute

$$T_i = T_{i-1} - \frac{(\alpha_t - \alpha_{t-1}) T_{i-1} \phi_i \phi_i^{\mathrm{T}} T_{i-1}}{1 + (\alpha_t - \alpha_{t-1}) \phi_i^{\mathrm{T}} T_{i-1} \phi_i};$$

   then,

$$P_t = T_{n+m+1} - \frac{T_{n+m+1} \varphi_{t-1} \varphi_{t-1}^{\mathrm{T}} T_{n+m+1}}{1 + \varphi_{t-1}^{\mathrm{T}} T_{n+m+1} \varphi_{t-1}}. \tag{4.1}$$

2. Compute the least-squares type estimate $\hat{\vartheta}_t$ according to the equation

$$\begin{aligned} \hat{\vartheta}_t = \hat{\vartheta}_{t-1} &+ P_t \varphi_{t-1} (y_t - \varphi_{t-1}^{\mathrm{T}} \hat{\vartheta}_{t-1}) \\ &+ P_t (\alpha_t - \alpha_{t-1})(\bar{\vartheta} - \hat{\vartheta}_{t-1}), \end{aligned} \tag{4.2}$$

   where

$$r_t = r_{t-1} + \|\varphi_{t-1}\|^2, \tag{4.3}$$

$$\alpha_t = (\log(r_t))^{1+\delta}, \quad \delta > 0. \tag{4.4}$$

3. If $\hat{\vartheta}_t \notin S(\bar{\vartheta}, r)$, project the estimate $\hat{\vartheta}_t$ onto the sphere $S(\bar{\vartheta}, r)$:

$$\bar{\vartheta}_t = \begin{cases} \hat{\vartheta}_t & \text{if } \hat{\vartheta}_t \in S(\bar{\vartheta}, r), \\ \frac{\hat{\vartheta}_t - \bar{\vartheta}}{\|\hat{\vartheta}_t - \bar{\vartheta}\|} r + \bar{\vartheta} & \text{otherwise.} \end{cases} \tag{4.5}$$

In Theorem 1 below we show that Eqs. (4.1)–(4.4) recursively compute the minimizer of a performance index of the form

$$\sum_{k=1}^{t} (y_k - \varphi_{k-1}^{\mathrm{T}} \vartheta)^2 + \alpha_t \|\vartheta - \bar{\vartheta}\|^2. \tag{5}$$

In view of this, an easy interpretation of the algorithm (4.1)–(4.5) is possible. In Eq. (5), the first term $\sum_{k=1}^{t} (y_k - \varphi_{k-1}^{\mathrm{T}} \vartheta)^2$ is the standard performance index for the least-squares algorithm, while the second term $\alpha_t \|\vartheta - \bar{\vartheta}\|^2$ penalizes those parameterizations which are far from the a priori nominal parameter value $\bar{\vartheta}$. In the performance index (5), a major role is played by the scalar function α_t which is aimed at providing a fair balancing between the penalized part and the least squares part of the performance index. This function should grow rapidly enough in order that the penalty for the estimates far away from the centre of the sphere can assert itself. On the other hand, the penalization term $\alpha_t \|\vartheta - \bar{\vartheta}\|^2$ should be mild enough to avoid destroying the closed-loop properties of the least squares algorithm. As a matter of fact, in Theorem 1 below, we show that the coefficient α_t in front of $\|\vartheta - \bar{\vartheta}\|^2$ grows rapidly enough so that term $\alpha_t \|\vartheta - \bar{\vartheta}\|^2$ asserts itself in such a way that in the long run the estimate $\hat{\vartheta}_t$ belongs to $S(\bar{\vartheta}, r)$. As a consequence, the projection operator in Eq. (4.5) is automatically switched off when t is large enough. The fact that the estimate becomes free of any projection in the long run, used in conjunction with the fact that the penalization term grows slowly enough, permits one to prove useful properties of our identification algorithm. In Theorem 2, we in fact show that $\bar{\vartheta}_t$ exhibits closed-loop properties which are similar to those of the standard recursive least squares estimate. These properties would be lost if the projection would not be switched off.

Theorem 1. (i) *The parameter estimate $\hat{\vartheta}_t$ obtained through the recursive procedure* (4.1)–(4.4) *initialized with*

$$\begin{aligned} &\hat{\vartheta}_0 = \bar{\vartheta}, \\ &r_0 = \mathrm{tr}(Q) \quad (Q = Q^{\mathrm{T}} > 0), \\ &P_0 = [Q + (\log(r_0))^{1+\delta} I]^{-1} \end{aligned} \tag{6}$$

is the minimizer of the performance index

$$D_t(\vartheta) = V_t(\vartheta) + \alpha_t \|\vartheta - \bar{\vartheta}\|^2, \tag{7}$$

where

$$V_t(\vartheta) = \sum_{k=1}^{t} (y_k - \varphi_{k-1}^{\mathrm{T}} \vartheta)^2 + (\vartheta - \bar{\vartheta})^{\mathrm{T}} Q (\vartheta - \bar{\vartheta}) \tag{8}$$

is the standard least-squares performance index with regularization term $(\vartheta - \bar{\vartheta})^{\mathrm{T}} Q (\vartheta - \bar{\vartheta})$ and

$$\alpha_t = \left(\log \left(\sum_{k=1}^{t} \|\varphi_{k-1}\|^2 + \mathrm{tr}(Q) \right) \right)^{1+\delta}. \tag{9}$$

(ii) *Assume that u_t is $\mathscr{F}_t$-measurable. Then, there exists a finite time instant $\bar{t}$ such that $\hat{\vartheta}_t \in S(\bar{\vartheta}, r)$, $t \geqslant \bar{t}$, almost surely.*

Proof. (i) It is not hard to see that $D_t(\vartheta)$ is a quadratic function of ϑ, whose minimizer $\hat{\vartheta}_t$ is given by

$$\hat{\vartheta}_t = \left[\sum_{k=1}^{t} \varphi_{k-1}\varphi_{k-1}^{\mathrm{T}} + Q + \alpha_t I\right]^{-1} \times \left[\sum_{k=1}^{t} y_k\varphi_{k-1} + (Q+\alpha_t I)\bar{\vartheta}\right].$$

In order to determine a recursive expression for $\hat{\vartheta}_t$, we define the matrix P_t as

$$P_t := \left[\sum_{k=1}^{t} \varphi_{k-1}\varphi_{k-1}^{\mathrm{T}} + Q + \alpha_t I\right]^{-1},$$

so that

$$\hat{\vartheta}_t = P_t\left[\sum_{k=1}^{t} y_k\varphi_{k-1} + (Q+\alpha_t I)\bar{\vartheta}\right]. \tag{10}$$

It is easy to show that the term $\sum_{k=1}^{t} y_k\varphi_{k-1}$ on the right-hand side of this last equation can be written as

$$\begin{aligned}\sum_{k=1}^{t} y_k\varphi_{k-1} &= y_t\varphi_{t-1} + \sum_{k=1}^{t-1} y_k\varphi_{k-1}\\ &= y_t\varphi_{t-1} + P_{t-1}^{-1}\hat{\vartheta}_{t-1} - (Q+\alpha_{t-1}I)\bar{\vartheta}.\end{aligned}$$

By substituting this last expression and the recursive expression for P_t^{-1} given by

$$P_t^{-1} = P_{t-1}^{-1} + \varphi_{t-1}\varphi_{t-1}^{\mathrm{T}} + (\alpha_t - \alpha_{t-1})I \tag{11}$$

in Eq. (10), we conclude that $\hat{\vartheta}_t$ can be determined as a function of the previous estimate $\hat{\vartheta}_{t-1}$ in the following way:

$$\begin{aligned}\hat{\vartheta}_t &= P_t\{y_t\varphi_{t-1} + [P_t^{-1} - \varphi_{t-1}\varphi_{t-1}^{\mathrm{T}}\\ &\quad - (\alpha_t - \alpha_{t-1})I]\hat{\vartheta}_{t-1} - (Q+\alpha_{t-1}I)\bar{\vartheta}\\ &\quad + (Q+\alpha_t I)\bar{\vartheta}\}\\ &= \hat{\vartheta}_{t-1} + P_t\varphi_{t-1}(y_t - \varphi_{t-1}^{\mathrm{T}}\hat{\vartheta}_{t-1})\\ &\quad + P_t(\alpha_t - \alpha_{t-1})(\bar{\vartheta} - \hat{\vartheta}_{t-1}),\end{aligned}$$

which is just the recursive expression of $\hat{\vartheta}_t$ in Eq. (4.2).

The fact that α_t given by Eq. (9) can be recursively computed through Eqs. (4.3) and (4.4) with the initialization $r_0 = \mathrm{tr}(Q)$ given in Eq. (6) is a matter of a simple verification.

Finally, the fact that step 1 in the algorithm actually computes the inverse of matrix P_t^{-1} given in Eq. (11) is a simple application of the matrix inversion lemma and is left to the reader. This completes the proof of (i).

(ii) Denote by $\hat{\vartheta}_t^{\mathrm{LS}}$ the minimizer of the least-squares performance index $V_t(\vartheta)$ and set

$$Q_t = \sum_{k=1}^{t} \varphi_{k-1}\varphi_{k-1}^{\mathrm{T}} + Q. \tag{12}$$

It is then easy to show that $\hat{\vartheta}_t = \arg\min_{\vartheta\in R^{n+m+1}} D_t(\vartheta)$ can be expressed as a function of $\hat{\vartheta}_t^{\mathrm{LS}}$ as follows:

$$\hat{\vartheta}_t = (Q_t + \alpha_t I)^{-1}Q_t\hat{\vartheta}_t^{\mathrm{LS}} + \alpha_t(Q_t+\alpha_t I)^{-1}\bar{\vartheta}.$$

By subtracting $\bar{\vartheta}$, we get

$$\begin{aligned}\hat{\vartheta}_t - \bar{\vartheta} &= (Q_t+\alpha_t I)^{-1}Q_t(\vartheta^\circ - \bar{\vartheta})\\ &\quad + (Q_t+\alpha_t I)^{-1}Q_t(\hat{\vartheta}_t^{\mathrm{LS}} - \vartheta^\circ).\end{aligned}$$

Thus, the norm of $\hat{\vartheta}_t - \bar{\vartheta}$ can be upper bounded as follows:

$$\begin{aligned}\|\hat{\vartheta}_t - \bar{\vartheta}\| &\leqslant \|\vartheta^\circ - \bar{\vartheta}\| + \|(Q_t+\alpha_t I)^{-1}Q_t^{1/2}\|\\ &\quad \times \|Q_t^{1/2}(\hat{\vartheta}_t^{\mathrm{LS}} - \vartheta^\circ)\|.\end{aligned} \tag{13}$$

We apply now Theorem 1 in [12] so as to upper bound the term $\|Q_t^{1/2}(\hat{\vartheta}_t^{\mathrm{LS}} - \vartheta^\circ)\|$. Since u_t is assumed to be $\mathscr{F}_t$-measurable, and also considering Assumption (A.1), by this theorem we obtain the following upper bound:

$$\|Q_t^{1/2}(\hat{\vartheta}_t^{\mathrm{LS}} - \vartheta^\circ)\|^2 = \mathrm{O}(\log(\mathrm{tr}(Q_t))), \quad \text{a.s.} \tag{14}$$

The term $\|(Q_t+\alpha_t I)^{-1}Q_t^{1/2}\|$ can instead be handled as follows.

Denote by $\{\lambda_{1,t},\ldots,\lambda_{n+m+1,t}\}$ the eigenvalues of the positive-definite matrix Q_t. Since Q_t is symmetric and positive-definite, there exists an orthonormal matrix T_t such that $Q_t = T_t\,\mathrm{diag}(\lambda_{1,t},\ldots,\lambda_{n+m+1,t})T_t^{-1}$ and $Q_t^{1/2} = T_t\,\mathrm{diag}(\lambda_{1,t}^{1/2},\ldots,\lambda_{n+m+1,t}^{1/2})T_t^{-1}$. Then,

$$\begin{aligned}&(Q_t+\alpha_t I)^{-1}Q_t^{1/2}\\ &\quad = T_t(T_t^{-1}(Q_t+\alpha_t I)T_t)^{-1}T_t^{-1}Q_t^{1/2}T_tT_t^{-1}\\ &\quad = T_t\,\mathrm{diag}\left(\frac{\lambda_{1,t}^{1/2}}{\lambda_{1,t}+\alpha_t},\ldots,\frac{\lambda_{n+m+1,t}^{1/2}}{\lambda_{n+m+1,t}+\alpha_t}\right)T_t^{-1}.\end{aligned}$$

This implies that

$$\|(Q_t+\alpha_t I)^{-1}Q_t^{1/2}\| = \max_{i=1,\ldots,n+m+1}\left(\frac{\lambda_{i,t}^{1/2}}{\lambda_{i,t}+\alpha_t}\right). \tag{15}$$

Consider now the function: $f(x)=x^{1/2}/(x+\alpha_t)$, $x\geqslant 0$. Such a function has an absolute maximum value $\frac{1}{2}\alpha_t^{-1/2}$ in $x=\alpha_t$. It then obviously follows from Eq. (15) that

$$\|(Q_t+\alpha_t I)^{-1}Q_t^{1/2}\|\leqslant \tfrac{1}{2}\alpha_t^{-1/2}. \tag{16}$$

Substituting the estimates (14) and (16) in Eq. (13), we obtain

$$\|\hat{\vartheta}_t-\bar{\vartheta}\|\leqslant\|\vartheta^\circ-\bar{\vartheta}\|+h_1\left(\frac{\log(\mathrm{tr}(Q_t))}{\alpha_t}\right)^{1/2},$$

h_1 being a suitable constant.

Observe now that from Eq. (3) it follows that $n_k^2\leqslant 2\max\{\|\vartheta^\circ\|^2,1\}[y_k^2+\|\varphi_{k-1}\|^2]$. Taking into account that the autoregressive part of model (3) is not trivial ($n>0$), this in turn implies that $n_k^2\leqslant 2\max\{\|\vartheta^\circ\|^2,1\}[\|\varphi_k\|^2+\|\varphi_{k-1}\|^2]$, from which it is easily shown that

$$\sum_{k=1}^{t} n_k^2\leqslant h_2\sum_{k=1}^{t+1}\|\varphi_{k-1}\|^2,$$

where h_2 is a suitable constant. From Assumption (A.2) and definition (12) of Q_t, we then get

$$\lim_{t\to\infty}\mathrm{tr}(Q_t)=\infty.$$

Since by definition (9), $\alpha_t=(\log(\mathrm{tr}(Q_t)))^{1+\delta}$, we then obtain that $\forall\varepsilon>0$ there exists a time instant τ such that $\|\hat{\vartheta}_t-\bar{\vartheta}\|\leqslant\|\vartheta^\circ-\bar{\vartheta}\|+\varepsilon$, $\forall t\geqslant\tau$. By Assumption (A.3), this implies that there exists a finite time instant $\bar{t}$ such that $\hat{\vartheta}_t\in S(\bar{\vartheta},r)$, $\forall t\geqslant\bar{t}$. This proves (ii). □

Part (ii) in Theorem 1 shows that $\hat{\vartheta}_t\in S(\bar{\vartheta},r)$, $t\geqslant\bar{t}$. This implies that the projection operation (4.5) is disconnected in the long run and, yet, the estimate lies inside the sphere $S(\bar{\vartheta},r)$. Since each model whose parameter belongs to $S(\bar{\vartheta},r)$ is controllable, from this the uniform controllability of the estimated model easily follows. In addition, thanks to the fact that the projection is disconnected, in Theorem 2 below we shall be able to show that the estimate $\bar{\vartheta}_t$ preserves closed-loop properties similar to those of the least-squares algorithm. The properties of the estimate $\bar{\vartheta}_t$ stated in Theorem 2 are fundamental for a successful application of our identification algorithm in adaptive control schemes suitable for possibly nonminimum-phase systems. In particular, property (i) is widely recognized as crucial for a correct selection of the control law, see e.g. [4, 5, 14, 15]. Securing property (ii) is important for obtaining stability and performance results, see e.g. [8, 7]. We also refer the reader to [19] for a pole placement application where properties (i) and (ii) have been exploited.

Recall that a standard measure of the controllability of model $y_t=\varphi_{t-1}^{\mathrm{T}}\vartheta+n_t$ is given by the absolute value of the determinant of the Sylvester matrix given by

$$\mathrm{Sylv}(\vartheta)=\underbrace{\begin{bmatrix} 1 & & & & & \\ -a_1 & 1 & & & & \\ -a_2 & -a_1 & \ddots & b_d & & \\ \vdots & -a_2 & \ddots\ 1 & b_{d+1} & \ddots & \\ -a_s & \vdots & -a_1 & \vdots & \ddots & b_d \\ & -a_s & -a_2 & b_s & & b_{d+1} \\ & & \ddots\ \vdots & & \ddots & \vdots \\ & & -a_s & & & b_s \end{bmatrix}}_{s\qquad\qquad s}, \tag{17}$$

where $s=\max\{n,d+m\}$ (see e.g. [9]). In particular $|\det(\mathrm{Sylv}(\vartheta))|\neq 0$ is equivalent to say that the model whose parameter is ϑ is controllable.

Theorem 2 (Properties of the estimate $\bar{\vartheta}_t$). (i) *There exists a constant $c>0$ such that $|\det(\mathrm{Sylv}(\bar{\vartheta}_t))|\geqslant c$, $\forall t$, almost surely.*

(ii) *Assume that u_t is $\mathscr{F}_t$-measurable. Then, the identification error satisfies the following bound*:

$$\|\vartheta^\circ-\bar{\vartheta}_t\|^2=\mathrm{O}\left(\frac{\left(\log\left(\sum_{k=1}^{t}\|\varphi_{k-1}\|^2\right)\right)^{1+\delta}}{\lambda_{\min}\left(\sum_{k=1}^{t}\varphi_{k-1}\varphi_{k-1}^{\mathrm{T}}\right)}\right), \quad \textit{almost surely.} \tag{18}$$

Proof. (i) Since the absolute value of the Sylvester matrix determinant is a continuous function of the system parameter ϑ and it is strictly positive for any $\vartheta\in S(\bar{\vartheta},r)$ (see Assumption (A.3)), we can take

$$c:=\min_{\vartheta\in S(\bar{\vartheta},r)}|\det(\mathrm{Sylv}(\vartheta))|>0. \tag{19}$$

Point (i) then immediately follows from the definition of $\bar{\vartheta}_t$ in Eq. (4.5).

(ii) Let us rewrite the performance index $D_t(\vartheta)$ as a function of the least-squares estimate $\hat{\vartheta}_t^{\mathrm{LS}}=$

$\arg\min_{\vartheta\in R^{n+m+1}} V_t(\vartheta)$:

$$D_t(\vartheta) = (\vartheta - \hat{\vartheta}_t^{\mathrm{LS}})^{\mathrm{T}}\left[\sum_{k=1}^{t}\varphi_{k-1}\varphi_{k-1}^{\mathrm{T}} + Q\right](\vartheta - \hat{\vartheta}_t^{\mathrm{LS}}) + \alpha_t\|\vartheta - \bar{\vartheta}\|^2 + V_t(\hat{\vartheta}_t^{\mathrm{LS}}).$$

From the definition of $\hat{\vartheta}_t$, it follows that

$$\begin{aligned}(\hat{\vartheta}_t - \hat{\vartheta}_t^{\mathrm{LS}})^{\mathrm{T}}&\left[\sum_{k=1}^{t}\varphi_{k-1}\varphi_{k-1}^{\mathrm{T}} + Q\right]\left(\hat{\vartheta}_t - \hat{\vartheta}_t^{\mathrm{LS}}\right) + \alpha_t\|\hat{\vartheta}_t - \bar{\vartheta}\|^2 \\ &\leqslant (\vartheta^\circ - \hat{\vartheta}_t^{\mathrm{LS}})^{\mathrm{T}}\left[\sum_{k=1}^{t}\varphi_{k-1}\varphi_{k-1}^{\mathrm{T}} + Q\right](\vartheta^\circ - \hat{\vartheta}_t^{\mathrm{LS}}) + \alpha_t\|\vartheta^\circ - \bar{\vartheta}\|^2 \\ &= \mathrm{O}(\alpha_t),\end{aligned} \tag{20}$$

almost surely, where the last equality is a consequence of the already cited Theorem 1 in [12] and of the boundedness of ϑ°. Consider now the equation

$$\begin{aligned}(\vartheta^\circ - \hat{\vartheta}_t)^{\mathrm{T}}&\left[\sum_{k=1}^{t}\varphi_{k-1}\varphi_{k-1}^{\mathrm{T}} + Q\right](\vartheta^\circ - \hat{\vartheta}_t) \\ \leqslant 2\Bigg\{&\left(\vartheta^\circ - \hat{\vartheta}_t^{\mathrm{LS}}\right)^{\mathrm{T}} \times \left[\sum_{k=1}^{t}\varphi_{k-1}\varphi_{k-1}^{\mathrm{T}} + Q\right]\left(\vartheta^\circ - \hat{\vartheta}_t^{\mathrm{LS}}\right) \\ &+ \left(\hat{\vartheta}_t^{\mathrm{LS}} - \hat{\vartheta}_t\right)^{\mathrm{T}}\left[\sum_{k=1}^{t}\varphi_{k-1}\varphi_{k-1}^{\mathrm{T}} + Q\right] \times \left(\hat{\vartheta}_t^{\mathrm{LS}} - \hat{\vartheta}_t\right)\Bigg\}.\end{aligned}$$

Since in view of Eq. (20) both terms on the right-hand side are almost surely $\mathrm{O}(\alpha_t)$, we get

$$(\vartheta^\circ - \hat{\vartheta}_t)^{\mathrm{T}}\left[\sum_{k=1}^{t}\varphi_{k-1}\varphi_{k-1}^{\mathrm{T}} + Q\right](\vartheta^\circ - \hat{\vartheta}_t) = \mathrm{O}(\alpha_t)$$

almost surely. From this,

$$\|\vartheta^\circ - \hat{\vartheta}_t\|^2 = \mathrm{O}\left(\frac{\alpha_t}{\lambda_{\min}\left(\sum_{k=1}^{t}\varphi_{k-1}\varphi_{k-1}^{\mathrm{T}} + Q\right)}\right), \text{ a.s.}$$

Since $\bar{\vartheta}_t = \hat{\vartheta}_t, \forall t \geqslant \bar{t}$ (point (ii) in Theorem 1) and also recalling definition (9) of α_t, point (ii) immediately follows. □

Remark 1. The convergence rate of the standard least-squares estimate $\hat{\vartheta}_t^{\mathrm{LS}}$ is given by ([12], Theorem 1),

$$\|\vartheta^\circ - \hat{\vartheta}_t^{\mathrm{LS}}\|^2 = \mathrm{O}\left(\frac{\log\left(\sum_{k=1}^{t}\|\varphi_{k-1}\|^2\right)}{\lambda_{\min}\left(\sum_{k=1}^{t}\varphi_{k-1}\varphi_{k-1}^{\mathrm{T}}\right)}\right).$$

This is slightly better than the bound (18) due to the exponent $1+\delta$ in this last bound. On the other hand, the least squares algorithm does not guarantee the estimated model controllability.

4. Conclusions

In the present contribution, we have introduced a new identification algorithm securing the estimated model controllability, which is widely recognized as a central problem in adaptive control. The proposed approach requires some a priori knowledge on the region to which the true parameter belongs, but, in contrast with the alternative stream of methods suitable for stochastic systems [4, 5, 13, 14], it has the main advantage to be easily implementable. It is therefore suggested as an effective solution to the controllability problem in all the situations in which the required a priori knowledge is available.

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